

# FINANCIAL RISK MANAGEMENT SPRING 2008

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# OVERVIEW

- ▶ The course offers an introduction into the evolving and expanding practice of financial risk management.
- ▶ Risk management is a complex process of identifying, quantifying and managing various risk exposures.
- ▶ Particular attention is devoted to the main risk management techniques such as Value at Risk (VaR), volatility and correlation models.

# SYLLABUS

## ▶ Market Risk

- ▶ Market Risk Measurement, Sources of Market Risk
- ▶ Hedging Linear Risk, Nonlinear Risk: Options
- ▶ Modeling Risk Factors, VaR Methods
- ▶ Volatility and Correlation Models

## ▶ Credit Risk

- ▶ Measuring Actuarial Default Risk, Measuring Default Risk from Market Prices
- ▶ Credit Exposure, Managing Credit Risk
- ▶ Credit Derivatives

## ▶ Operational Risk and Regulation

- ▶ Operational Risk, Risk Capital and RAROC
- ▶ The Basel Accord

# GENERAL INFORMATION

- ▶ Class Attendance: Highly encouraged
- ▶ Textbooks:
  - ▶ Philippe Jorion (2009), “Financial Risk Manager Handbook,” Fifth Edition, GARP, Wiley Finance
  - ▶ John Hull (2008), *Options, Futures and Other Derivatives*, 6<sup>th</sup> Edition, Prentice Hall
- ▶ Recommended books:
  - ▶ Peter Christoffersen (2003), “Elements of Financial Risk Management,” Academic Press

# COURSE EVALUATION

- ▶ Homework 1: 15%
- ▶ Homework 2: 15%
- ▶ Take-Home Midterm Exam: 35%
- ▶ Final Exam: 35%

# PREREQUISITES

- ▶ This is a technical course and requires a good understanding of calculus, probability and statistics.
- ▶ Please read:
  - ▶ Chapter 2: Fundamentals of Probability
  - ▶ Chapter 3: Fundamentals of Statistics
  - ▶ Chapter 4: Monte Carlo Methods
- ▶ Prerequisite courses:
  - ▶ Derivative Securities (course # 756.761)
  - ▶ Investment Analysis and Portfolio Management (course # 756.760)
- ▶ Please read:
  - ▶ Chapter 5-9

# RISK AND UNCERTAINTY

- ▶ Risk and uncertainty have a rather short history in economics
- ▶ The *formal* incorporation of risk and uncertainty into economic theory was only accomplished in 1944, when John Von Neumann and Oskar Morgenstren published their *Theory of Games and Economic Behavior*
- ▶ The very *idea* that risk and uncertainty might be relevant for economic analysis was only really suggested in 1921, by Frank H. Knight in his formidable treatise, *Risk, Uncertainty and Profit*.

# RISK AND UNCERTAINTY

- ▶ Indeed, he linked profits, entrepreneurship and the very existence of the free enterprise system to risk and uncertainty.
- ▶ Much has been made of Frank H. [Knight](#)'s (1921: p.20, Ch.7) famous distinction between "risk" and "uncertainty". In Knight's interpretation, "risk" refers to situations where the decision-maker can assign mathematical probabilities to the randomness which he is faced with. In contrast, Knight's "uncertainty" refers to situations when this randomness "cannot" be expressed in terms of specific mathematical probabilities. As John Maynard Keynes was later to express it:

*"By 'uncertain' knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty... The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence... About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know." (J.M. [Keynes](#), 1937)*

# RISK AND UNCERTAINTY

- ▶ Nonetheless, many economists dispute this distinction, arguing that Knightian risk and uncertainty are one and the same thing. For instance, they argue that in Knightian uncertainty, the problem is that the agent *does not* assign probabilities, and not that she actually *cannot*, i.e. that uncertainty is really an epistemological and not an ontological problem, a problem of "knowledge" of the relevant probabilities, not of their "existence".
- ▶ Going in the other direction, some economists argue that there are actually no probabilities out there to be "known" because probabilities are really only "beliefs". In other words, probabilities are merely subjectively-assigned expressions of beliefs and have no necessary connection to the true randomness of the world (if it is random at all!).

# RISK AND UNCERTAINTY

- ▶ Nonetheless, some economists, particularly Post Keynesians such as G.L.S. Shackle (1949, 1961, 1979) and Paul Davidson (1982, 1991) have argued that Knight's distinction is crucial. In particular, they argue that Knightian "uncertainty" may be the only relevant form of randomness for economics - especially when that is tied up with the issue of time and information. In contrast, situations of Knightian "risk" are only possible in some very contrived and controlled scenarios when the alternatives are clear and experiments can conceivably be repeated -- such as in established gambling halls. Knightian risk, they argue, has no connection to the murkier randomness of the "real world" that economic decision-makers usually face: where the situation is usually a unique and unprecedented one and the alternatives are not really all known or understood. In these situations, mathematical probability assignments usually cannot be made. Thus, decision rules in the face of uncertainty ought to be considered different from conventional expected utility.

# RISK AND UNCERTAINTY

- ▶ The "risk versus uncertainty" debate is long-running and far from resolved at present. As a result, we shall attempt to avoid considering it with any degree of depth here. What we shall refer throughout as "uncertainty" does not correspond to its Knightian definition. Instead, we will use the term risk and uncertainty interchangeably.

# RISK AND UNCERTAINTY

- ▶ After Knight, economists finally began to take it into account: John Hicks (1931), John Maynard Keynes (1936, 1937), Michal Kalecki (1937), Helen Makower and Jacob Marschak (1938), George J. Stigler (1939), Gerhard Tintner (1941), A.G. Hart (1942) and Oskar Lange (1944), appealed to risk or uncertainty to explain things like profits, investment decisions, demand for liquid assets, the financing, size and structure of firms, production flexibility, inventory holdings, etc.
- ▶ The great barrier in a lot of this early work was in making precise what it means for "uncertainty" or "risk" to affect economic decisions.
- ▶ How do agents evaluate ventures whose payoffs are random? How exactly does increasing or decreasing uncertainty consequently lead to changes in behavior?
- ▶ These questions were crucial, but with several fundamental concepts left formally undefined, appeals risk and uncertainty were largely of a heuristic and unsystematic nature.

# RISK AND UNCERTAINTY

- ▶ The great missing ingredient was the formalization of the notion of "choice" in risky or uncertain situations.
- ▶ Already Hicks (1931), Marschak (1938) and Tintner (1941) had a sense that people should form preferences over distributions, but how does one separate the element of attitudes towards risk or uncertainty from pure preferences over outcomes?
- ▶ Alternative hypotheses included ordering random ventures via their means, variances, etc., but no precise or satisfactory means were offered up.

# RISK AND UNCERTAINTY

- ▶ Surprisingly, Daniel Bernoulli's (1738) notion of expected utility which decomposed the valuation of a risky venture as the sum of utilities from outcomes weighted by the probabilities of outcomes, was generally not appealed to by these early economists.
- ▶ Part of the problem was that it did not seem sensible for rational agents to maximize expected utility and not something else.
- ▶ Specifically, Bernoulli's assumption of diminishing marginal utility seemed to imply that, in a gamble, a gain would increase utility less than a decline would reduce it. Consequently, many concluded, the willingness to take on risk must be "irrational", and thus the issue of choice under risk or uncertainty was viewed suspiciously, or at least considered to be outside the realm of an economic theory which assumed rational actors.

# RISK AND UNCERTAINTY

- ▶ The great task of John von Neumann and Oskar Morgenstern (1944) was to lay a rational foundation for decision-making under risk according to expected utility rules.
- ▶ The novelty of using the axiomatic method - combining sparse explanation with often obtuse axioms - ensured that most economists of the time would find their contribution inaccessible and bewildering.
- ▶ Indeed, there was substantial confusion regarding the structure and meaning of the von Neumann- Morgenstern expected utility itself.

# RISK AND UNCERTAINTY

- ▶ In the von Neumann-Morgenstern hypothesis, probabilities are assumed to be "*objective*" or exogenously given by "Nature" and thus cannot be influenced by the agent. However, the problem of an agent under uncertainty is to *choose* among lotteries, and thus find the "best" lottery in  $\Delta(X)$ , where  $\Delta(X)$  is the set of simple lotteries on  $X$  (outcomes). One of von Neumann and Morgenstern's major contributions to economics more generally was to show that if an agent has preferences defined over lotteries, then there is a utility function  $U: \Delta(X) \rightarrow \mathbb{R}$  that assigns a utility to every lottery  $p \in \Delta(X)$  that represents these preferences.

# EXPECTED UTILITY FUNCTION

- ▶ The study of decision making under uncertainty is a vast subject
- ▶ However, financial applications almost invariably proceed under the guise of the expected utility hypothesis: people rank random prospects according to the expected utility of those prospects.
- ▶ Analytically, this involves solving problems requiring selecting choice variables to maximize an expected utility function.

$$EU(x) = \sum_{j=1}^S p_j U(x_j)$$

# EXPECTED UTILITY FUNCTION

where  $EU(x)$  is the expected utility of  $x$ ;  $S$  is the number of possible future state of the world;  $p_j$  is the probability that state  $j$  occur; and  $U(x_j)$  is the utility associated with the amount of  $x$  received in state  $j$ .

- ▶ Expected utility function ( $E U$ ) ranks risky prospects with an ordering that is unique up to linear transformation.
- ▶ *However*, how are we going to assign probabilities? Is it subjective or objective. CAPM model, for example treats these probabilities as objective by assuming that expectations and/or individuals are homogenous.

# EXPECTED UTILITY FUNCTION

- ▶ Such an assumption might be understandable in general equilibrium framework.
- ▶ But the decision problems encountered are partial equilibrium.
- ▶ Consider the problem of determining the cost of the risk. Economic agent's choice is between buying an insurance or investing in a risky capital project. Let the expected value of one period ahead wealth be  $E(W_{t+1}) = \Omega$ . Observe that  $\Omega$  is a parameter that permits the certainty equivalent income of a risky prospect to be defined as

$\Omega - C$ , where  $C$  is the cost of risk. It follows from the expected utility axioms that the cost of risk can be calculated as the difference between the expected value of the risky prospect and the associated certainty equivalent income:

# EXPECTED UTILITY FUNCTION

$$U[\Omega - C] = \sum_{j=1}^S p_j U[W_j] = EU[W_{t+1}]$$

We can estimate cost of risk by expanding  $U[\Omega - C]$  around  $\Omega$  the first order approximation is

$$U[\Omega - C] = U[\Omega] + U'[\Omega](\Omega - C - \Omega) = U[\Omega] - U'[\Omega]C$$

Similarly, the second order approximation for the function  $U(W_{t+1})$  is

$$U[W_{t+1}] = U[\Omega] - U'[\Omega](W_{t+1} - \Omega) + \frac{1}{2} U''[\Omega](W_{t+1} - \Omega)^2$$

# EXPECTED UTILITY FUNCTION

$$EU[W_{t+1}] = U[\Omega] + \frac{1}{2} U''[\Omega] \sigma_{W_{t+1}}^2$$

Remember

This gives us  $U[\Omega - C] = EU[W_{t+1}]$

$$C = -\frac{U''[\Omega]}{2U'[\Omega]} \sigma_{W_{t+1}}^2 \xrightarrow{\text{yields}} \frac{C}{W_t} = -\frac{U''[\Omega]W_t}{2U'[\Omega]} \sigma_{(1+R)}^2$$

This shows us that the cost of risk will vary across utility functions. This result also provides a theoretical measure of the risk. The measure of absolute risk aversion, the relative risk aversion as well as the variance of the interest rate have an effect on the cost of risk.

# EXPECTED UTILITY FUNCTION

- ▶ Here using Taylor expansion, we present mean-variance optimization problem. Utility depends positively on the terminal wealth (maybe interpreted as return on the asset) but negatively on the variance of terminal wealth. Risk averse individual prefers higher return but lower risk. If we introduce third moment preference (skewness), we will be talking about mean-variance-skewness optimization problem.
- ▶ Question: What will be third order approximation if you expand the function  $U(W_{t+1})$  around the terminal wealth? Why do we require  $U' > 0$ ,  $U'' < 0$ , and  $U''' > 0$ ? What are the economic meanings of these derivatives?

# EXPECTED UTILITY FUNCTION

- ▶ Mean-Variance Optimization:

$$EU_{MV}[W_{t+1}] = U[\Omega] - a\sigma_{W_{t+1}}^2$$

- ▶ Mean-Variance-Skewness Optimization

$$EU_{MVS}[W_{t+1}] = U[\Omega] - a\sigma_{W_{t+1}}^2 + b \text{skew}(W_{t+1})$$

# EXAMPLE: PORTFOLIO DIVERSIFICATION

- ▶ Consider now set of outcomes as price of an asset, lotteries as different assets, which asset class you will invest?
- ▶ It depends on several factors:
  - ▶ Return on the asset
  - ▶ Riskiness of the asset
- ▶ When we speak of the riskiness of an asset, we are speaking of the volatility of the control over resources that is induced by holding that asset. From the perspective of a consumer, concern focuses on how holding an asset affects the consumer's purchasing power.
- ▶ There are many possible sources of asset riskiness. For now we focus on currency risk. That is, we focus on how currency denomination alone affects riskiness. For example, we may think of debt issued in two different currency denominations by the U.S. government, so that the only clear difference in risk characteristics derives from the difference in currency denomination.

# PORTFOLIO DIVERSIFICATION

- ▶ The basic sources of risk from currency denomination are exchange rate risk and inflation risk. Exchange rate risk is the risk of unanticipated changes in the rate at which a currency trades against other currencies. Inflation risk is the risk of unanticipated changes in the rate at which a currency trades against goods priced in that currency.

# PORTFOLIO DIVERSIFICATION

- ▶ If we consider the uncovered real return from holding a foreign asset, it is

$$r_f = i^* + \Delta s - \pi$$

- ▶ So if  $\Delta s$  and  $\pi$  are highly correlated, the variance of the real return can be small in principle, even smaller than the variance of the return on the domestic asset. Thus in countries with very unpredictable inflation rates, we can see how holding foreign assets may be less risky than holding domestic assets. This can be the basis of capital flight capital outflows in response to increased uncertainty about domestic conditions. Capital flight can simply be the search for a hedge against uncertain domestic inflation.

# PORTFOLIO DIVERSIFICATION

- ▶ The notion of the riskiness of an asset is a bit tricky: it always depends on the portfolio to which that asset will be added. Similarly, the risk of currency denomination cannot be considered in isolation. That is, we cannot simply select a currency and then determine its riskiness. We need to know how the purchasing power of that currency is related to the purchasing power of the rest of the assets we are holding. The riskiness of holding a Euro denominated bond, say, cannot be determined without knowing its correlation with the rest of my portfolio.

# PORTFOLIO DIVERSIFICATION

- ▶ We will use correlation as our measure of relatedness. The correlation coefficient between two variables is one way to characterize the tendency of these variables to move together. An asset return is positively correlated with my portfolio return if the asset tends to gain purchasing power along with my portfolio. An asset that has a high positive correlation with my portfolio is risky in the sense that buying it will increase the variance of my purchasing power. Such an asset must have a high expected rate of return for me to be interested in holding it.

# PORTFOLIO DIVERSIFICATION

- ▶ In contrast, adding an asset that has a low correlation with my portfolio can reduce the variance of my purchasing power. For example, holding two equally variable assets that are completely uncorrelated will give me a portfolio with half the variability of holding either asset exclusively. When one asset declines in value, the other has no tendency to follow suit. In this case diversification “pays”, in the sense that it reduces the riskiness of my portfolio.

# PORTFOLIO DIVERSIFICATION

- ▶ From the point of view of reducing risk, an asset that is negatively correlated with my portfolio is even better. In this case there is a tendency of the asset to offset declines in the value of my portfolio. That is, when the rest of my portfolio falls in value, this asset tends to rise in value. If two assets are perfectly negatively correlated, we can construct a riskless portfolio by holding equal amounts of each asset: whenever one of the assets is falling in value, the other is rising in value by an equal amount. In order to reduce the riskiness of my portfolio, I may be willing to accept an inferior rate of return on an asset in order to get its negative correlation with my portfolio rate of return.

# PORTFOLIO DIVERSIFICATION

- ▶ If we look at an asset in isolation, we can determine its expected return and the variance of that return. A high variance would seem on the face of it to be risky. However we have seen that the currency risk and inflation risk of an isolated asset are not very interesting to consider. We may be interested in holding an asset denominated in a highly variable foreign currency if doing so reduces the variance of our portfolio rate of return. To determine whether the asset can do this, we must consider its correlation with our current portfolio. A low correlation offers an opportunity for diversification, and a negative correlation allows even greater reductions in portfolio risk. We are willing to pay extra for this reduction in risk, and the risk premium is the amount extra we pay. If adding foreign assets to our portfolio reduces its riskiness, then the risk premium on domestic assets will be positive.

# OPTIMAL DIVERSIFICATION

- ▶ Consider an investor who prefers higher average returns but lower risk. We will capture these preferences in a utility function, which depends positively on the average return of the investors portfolio and negatively on its variability,  $U(E(r_p); \text{var}(r_p))$ . We can think of portfolio choice as a two stage procedure. First we determine the portfolio with the lowest risk: the minimum-variance portfolio. Second, we decide how far to deviate from the minimum-variance portfolio based on the rewards to risk bearing.

- ▶ Let us return to our investor who prefers higher average returns but lower risk, as represented by the utility function  $U(E(rp); \text{var}(rp))$ . Domestic assets pay  $r = i - \pi$  and foreign assets pay  $r_f = i^* + \Delta s - \pi$  as real returns to domestic residents. The total real return on the portfolio  $r_p$  will then be a weighted average of the returns on the two assets, where the weight is just  $\alpha$  (the fraction of the portfolio allocated to foreign assets).

$$r_p = \alpha r_f + (1 - \alpha)r$$

- ▶ Therefore the expected value of the portfolio rate of return is

$$E(r_p) = \alpha E(r_f) + (1 - \alpha)E(r)$$

# OPTIMAL DIVERSIFICATION

- ▶ And the variance of the portfolio

$$\text{Var}(r_p) = \alpha^2 \text{var}(r_f) + 2\alpha(1 - \alpha) \text{cov}(r, r_f) + (1 - \alpha)^2 \text{var}(r)$$

Consider how to maximize utility, which depends on the mean and variance of the portfolio rate of return. The objective is to choose  $\alpha$  to maximize utility.

$$\text{Max}_{\alpha} U(\alpha E(r_f) + (1 - \alpha)E(r), \alpha^2 \text{var}(r_f) + 2\alpha(1 - \alpha) \text{cov}(r, r_f) + (1 - \alpha)^2 \text{var}(r))$$

$$\frac{dU}{d\alpha} = [E(r_f) - E(r)]U_1 + 2[\alpha \text{var}(r_f) - (1 - \alpha)\text{var}(r) + (1 - 2\alpha) \text{Cov}(r, r_f)]U_2$$

# OPTIMAL DIVERSIFICATION

- ▶ As long as this derivative is positive, so that increasing  $\alpha$  produces and increase in utility, we want to increase alpha. If this derivative is negative, we can increase utility by reducing alpha. These considerations lead to the “first-order condition”: the requirement that  $dU/d\alpha = 0$  at a maximum. We use the first-order condition to produce a solution for  $\alpha$ .

$$\alpha = \frac{[E(rf) - E(r)] - \frac{2U_2}{U_1} [\text{var}(r) - \text{Cov}(r, rf)]}{-\frac{2U_2}{U_1} [\text{var}(rf) + \text{var}(r) - 2\text{Cov}(r, rf)]}$$

# OPTIMAL DIVERSIFICATION

- ▶ Here  $RRA = -2U_2/U_1$  (the coefficient of relative risk aversion) and  $\sigma^2 = \text{var}(rf) + \text{var}(r) - 2\text{cov}(r,rf)$ .

$\sigma^2 =$

- ▶ Recalling that  $E(rf) - E(r) = i^* + \Delta s^e - i = rp$ ;

we therefore have

$$\alpha = -\frac{rp}{RRA \sigma^2} + \alpha_{minvar}$$

- ▶ Here

$$\alpha_{minvar} = [\text{var}(r) - \text{Cov}(r,rf)]/\sigma^2$$

is the  $\alpha$  that yields the minimum variance portfolio (Kouri 1978), so the rest can be considered the speculative portfolio share. Investors can be thought of as initially investing entirely in the minimum variance portfolio and then exchanging some of the lower return asset for some of the higher return asset. They accept some increase in risk for a higher average return. If the assets have the same expected return, they will simply hold the minimum variance portfolio.

# RISK MANAGEMENT

- ▶ Firms and individuals face an array of risks that have to be managed.
- ▶ **Market Risk**
  - ▶ Risk to a financial portfolio from movements in market prices such as equity prices, FX, interest rates and commodity prices
- ▶ **Credit and Liquidity Risk**
  - ▶ It consists of risk of default (complete or partial, maturity), costs associated with having to unwind position, and the possibility that credit lines may be restricted
- ▶ **Operational Risk**
  - ▶ Risk of loss due to physical catastrophe, technical failure, human error including fraud, failure of management and process error
- ▶ **Business (Commercial) Risk**
  - ▶ Business cycle, competitive behavior, technology

# RISK MANAGEMENT

- ▶ Question arises: Under what condition can each of these risks be managed independently of the other types of risks?
- ▶ Is it possible to separate production decision from risk management decision? If possible, then it is possible to use risk management process that considers the problem of managing market risks independently from general business risks.
- ▶ Consider a farmer, who uses futures market to lock in the price that is received for the crop at harvest.

# RISK MANAGEMENT

- ▶ Does that mean profit to be determined by factors influencing the yield per acre? Or profit would also be affected by the hedging decision? Since hedge position can lose money as well as make money, all hedging decision has also speculative features.
- ▶ If the hedge loses money, then profits will be increased by not hedging. If the price at harvest is higher than locked price, then the hedger loses money. Therefore, optimal hedge has to take expected future price into account.

# RISK MANAGEMENT: CAPM VIEW

▶ CAPM:  $E(R_i) = R_f + \beta_i(E(R_M) - R_f)$

Asset-specific risk can be eliminated by **DIVERSIFICATION**

- ▶ Asset-specific risk can be avoided via diversification. Having exposure to it is not rewarded in the market. (Why would one expect to be compensated for needlessly carrying diversifiable risk?) Under these circumstances only the non-diversifiable risk of an asset would be rewarded.
- ▶ Investors should hold a combination of the risk free asset and market portfolio (which depends on the investor's risk aversion)
- ▶ It is important to note that the model is a normative equilibrium model; i.e. how the world should behave if all investors are rational and market are efficient.

# RISK MANAGEMENT: CAPM VIEW

- ▶ In this setup, companies are wasting their time on FRM and have no socially redeeming value. Indeed, if the world would behave exactly as suggested by CAPM
  - ▶ All investors should hold the same portfolio (market portfolio), which is perfectly diversified.
  - ▶ The knowledge of beta is enough to determine the asset expected return.

# RISK MANAGEMENT: WHY? (PART 2)

- ▶ Modigliani-Miller Theorem (Proposition I)

$$V = (E + D) = \text{NOI} \div r \quad (\text{perpetuity})$$

Firms should maximize expected profits!

The value of a firm is independent of its risk structure

# WHY SHOULD COMPANIES MANAGE RISK?

- ▶ Bankruptcy costs
  - ▶ FRM can increase the value of the firm by reducing the probability of default
- ▶ Taxes
  - ▶ FRM can help reduce taxes by reducing the volatility of earnings
- ▶ Capital structure and cost of capital
  - ▶ Major source of corporate default is the inability to service debt. FRM can help the firm to have a higher debt-to-equity ratio, which is beneficial if debt is inexpensive
- ▶ Compensation package
  - ▶ The riskier the firm, the more compensation current and potential employees will require. FRM might reduce the costs of retaining and recruiting key personnel.

# FRM: SOME EVIDENCE

- ▶ In 1998, Wharton surveyed 2,000 companies. Only 400 responded
- ▶ Half of the respondents reported that they use derivatives for FRM
- ▶ One-third of derivative users, often take positions according to their market views → they may be using derivatives to increase risk!
- ▶ Large firms tend to manage risk more actively than small firms

# DOES FRM IMPROVE FIRM PERFORMANCE?

## ▶ Yes!

- ▶ Gold Mining industry: share prices were less sensitive to gold price movements if FRM implemented
- ▶ Similar results in natural gas industry
- ▶ In general, it has been found that FRM reduces exposure to interest rate and exchange rate movements
- ▶ Less volatile cash flows result in lower cost of capital and more investment

## ▶ Be careful!!!

# FINANCIAL DISASTERS

- ▶ Felix Rohatyn (Wall Street)  
“26-year olds with computers are creating financial hydrogen bombs”
- ▶ Barings, Metallgesellschaft, Orange County, Daiwa, SocGen:  
They have one element in common, poor management of financial risk
- ▶ Derivatives are very effective tools to hedge and speculate but they can lead to large losses if used inappropriately
- ▶ From 1987 to 1998 losses attributed to derivatives totaled \$28B  
Market size \$90 trillion → 0.03% of losses

# LOSSES ATTRIBUTED TO DERIVATIVES: 1993 - 1999

Company	Date	Instrument	Loss (\$ million)
Orange County, US	1994	Repos	1,810
Showa Shell, Japan	1993	Currency Forwards	1,580
Kashima Oil, Japan	1994	Currency Forwards	1,450
Metallgesellschaft	1994	Oil Futures	1,340
Barings, UK	1995	Stock Index Futures	1,330
Ashanti, Ghana	1999	Gold Exotic	570
Yakult Honsha, Japan	1998	Stock Index Derivatives	523
Codelco, Chile	1994	Copper Futures	200
NatWest, UK	1997	Swaption	127
Procter & Gamble, US	1994	Swaps	157
Amaranth	2006	Natural Gas Futures	7000

# BARINGS

- ▶ February 26, 1995
- ▶ 233 year old bank
- ▶ 28 year old Nick Leeson
- ▶ \$1,300,000,000 loss
- ▶ Bought by ING for \$1.5

# METALLGESELLSHAFT

- ▶ 14th largest industrial group in Germany
- ▶ 58,000 employees
- ▶ offered long term oil contracts
- ▶ hedge by long-term forward contracts
- ▶ short term contracts were used (rolling hedge)
- ▶ 1993 price fell from \$20 to \$15
- ▶ \$1B margin call in cash

# ORANGE COUNTY

- ▶ Bob Citron, the county treasurer
- ▶ \$7.5B portfolio (schools, cities)
- ▶ Borrowed \$12.5B, invested in 5yr. notes
- ▶ Interest rates increased
- ▶ Reported at cost - **big mistake!**
- ▶ Realized loss of \$1.64B

# DAIWA

- ▶ 12-th largest bank in Japan
- ▶ September 1995
- ▶ Hidden loss of \$1.1B accumulated over 11 years
- ▶ Toshihide Igushi, trader in New York
- ▶ Had control of front and back offices
- ▶ In 92 and 93 FED warned Daiwa about bad management structure

# INTRODUCTION TO HEDGING WITH FUTURES AND OPTIONS





# WHAT ARE FUTURES & OPTIONS?

- ▶ Futures contracts are standardized, legally binding agreements to buy or sell a specific product or financial instrument in the future. The buyer and seller of a futures contract agree on a price today for a product to be delivered or settled in cash at a future date. Each contract specifies the quantity, quality and the time and location of delivery and payment.
- ▶ Options can be thought of as insurance policies. The option buyer pays a price for the right – but not the obligation – to buy or sell a futures contract within a stated period of time at a predetermined price.

# CONTRACT TRADE POSITIONS

Futures Contract:

- ▶ An obligation to buy or sell a commodity that meets set grades and standards on some future date.

Types	Terminology	Price Advantage	Deliver/Offset Cash Settle
Sell	Short	Prices down 	Long
Buy	Long	Prices up 	Short

# HOW ARE PRICES DETERMINED?

Actions of hedgers and speculators looking at:

- ▶ Supply
- ▶ Demand
- ▶ Political factors
- ▶ Psychological factors
  
- ▶ And coping with price volatility

# PRICE FLUCTUATIONS

Minimum (tick): 2 ½ cents/cwt.



- ▶ Each tick move: 400 cwt. X .025 cwt.  
= **\$10.00 per Lean Hog contract**

Maximum daily limit from previous day's settlement price :

- ▶ Lean Hogs = \$3.00 x 400 cwt. = \$1,200

# MARKET SAFEGUARDS

## Clearing House:

- ▶ All the clearing firms of the Exchange together stand in between every buyer and seller.

## Market-to-Market:

- ▶ Funds move into or out of each trading account daily, based on settlement price change.
- ▶ Sell (short) one Lean Hog contract with \$1.00 price move:

**77 (\$400 out of account)**

**76**



**75 (\$400 in account)**

- ▶ Vice-versa in long contracts

# MARKET SAFEGUARDS - PERFORMANCE BOND OR MARGIN

Like a Security Deposit. Must be maintained at a certain level while futures position held. Usually less than 10% of contract value.

Live Cattle cash value vs. Futures contract margin example:

$$40,000\# = 400 \text{ cwt.}$$

$$400 \text{ cwt.} \times \$90 = \$36,000$$

$$\$800/\$36,000 = 4.5\%$$

	Live Cattle	Corn
Hedge	800	800
Speculative	1,080	1,080



# PERFORMANCE BOND EXAMPLE – CATTLE

Contract Size  $\frac{40,000 \text{ lbs.}}{100} = 400 \text{ Cwt.}$

Minimum Tick (\$.025/Cwt.)  $\frac{400 \text{ Cwt.}}{100} = 10.00$

Limit Move (\$3.00/Cwt.)  $\frac{400 \text{ Cwt.}}{100} = 1,200$

Performance Bond:  $\$1,080$  (initial)

Maintenance  $\$800$

Day (Trade Price)	Futures Price (Buy/Sell)	Dollar Change	Account Balance
Trade Day	Sell LC 90.50		2,000
LC Close 90.000		$+.50 \times 400 = +200$	2,200
LC Close 92.50		$-2.50 \times 400 = -1,000$	1,200
LC Close 93.525		$-1.025 \times 400 = -410$	790 (MC= 290)
LC Close 91.525		$+2.00 \times 400 = +800$	1,880
Offset LC	Buy LC 90.10	$+1.42 \times 400 = +570$	2,450
<b>Trade Summary</b>		<b>Account Summary</b>	
S 90.50		1,080 Initial +290 MC = 1,370	
B 90.10 = $+.40 \times 400 = \$160$		Trade balance = $2,000 - 2,450 = \$450$	
		160 Profit + 290 MC = \$450	

# HEDGING (RISK SHIFTING)

## Definition of Hedging:

- Taking an opposite position in futures from your present cash position.

Cash	Futures
<b>Owner of inventory</b> <ul style="list-style-type: none"><li>• <b>Risk in down</b> markets</li></ul>	<b>Seller of contracts (Short)</b> <ul style="list-style-type: none"><li>• <b>Gain in down</b> markets</li></ul>
<b>User of inventory</b> <ul style="list-style-type: none"><li>• <b>Risk in up</b> markets</li></ul>	<b>Buyer of contracts (Long)</b> <ul style="list-style-type: none"><li>• <b>Gain in up</b> markets</li></ul>

# MARKET MAKE-UP

## Commercial Hedgers

<b>Segment</b>	<b>Definition</b>
<b>Livestock producers</b>	Raise hogs that are processed into pork products
<b>Meat packers</b>	Slaughter and process livestock into meat products which they market to wholesale and retail distributors
<b>Food companies</b>	Manufacturing firms, wholesale and retail distributors

# MARKET MAKE-UP

## Discretionary traders

Segment	Definition
<b>Institutional Managed Money</b>	<ul style="list-style-type: none"><li>• Commodity Trading Advisors (CTAs)</li><li>• Hedge Funds</li><li>• Commodity Indexes (GSCI, Rodgers Commodity Funds, Dow Jones/AIG)</li></ul>
<b>Individual traders</b>	Locals and brokers (pit and screen)
<b>Arcades/ Prop Shops</b>	Boutique trading firms

# WHY FIRMS USE COMMODITY F & O

Lock in or cap input costs

Protect inventory

Offer customers fixed price contracts

Reduce risk of regular seasonal moves

Minimize damage from long-term cyclical moves

Cover opportunity losses from forward pricing

Control risks in other parts of business

Capture windfall pricing opportunities

# HEDGING WITH FUTURES AND OPTIONS

- ▶ Hedging consists of defining:
  - ▶ Your open cash pricing risk,
  - ▶ The corresponding futures contract to offset the risk,
  - ▶ The futures contract month closest to cash risk production/procure period in question,
  - ▶ An estimated basis in the marketing/ procure month in order to project a hedged price.
  - ▶ Plus, firm's hedge policy plans; such as price objectives, percentages to hedge, and the offset policy to meet overall risk management objectives.

## **Basic approaches to hedging price risk:**

Buy (long) hedging – Procurement

Sell (short) hedging – Production

# SELL FUTURES TO HEDGE INVENTORY



# BUY A PUT TO PROTECT A PRICE LEVEL



# HEDGING AND BASIS

Basis and Expected Hedged (target) Price:

- ▶ Futures contracts are standardized as to size, quality, etc.; thus a zero basis never exists because the underlying cash (spot) price varies based on quality (premiums & discounts) and location factors effecting supply and demand.
- ▶ Futures prices are quoted daily. B/S Hedgers can convert the futures price to a hedged price return by adjusting for their historical basis in the marketing month in question.
- ▶ Actual Basis derived when hedge is lifted.

$$\text{Fut./Mo./ Price} \quad \pm \quad \text{Est. Local Basis (Ave.)} \quad = \quad \text{Expected Hedge Price}$$

# LEAN HOG/PRODUCER – ZERO BASIS RISK

Hogs hedged at 75.00 and zero basis projection at the time the hedge was initiated should equate to a final hedged price of \_\_\_\_\_ (+/- \$ 75.00) = 0  
75.00

Sale Month = June Historical June Basis Est. = \_\_\_\_\_ 0

Sell Futures 75.00 +/- 0 Basis = 75.00  
(expected hedged price)

# LEAN HOG/PRODUCER – ZERO BASIS EXAMPLE

Lower Prices:

Cash price	Futures Price	Basis	Expected Hedge Price
	Sell <u>75.00</u>	<u>0</u> (estimated)	( <u>75.00</u> )
<u>67.50</u> (sell cash)	Buy <u>67.50</u>	<u>0</u> (actual)	
<u>67.50</u>	+ <u>7.50</u>	=	Actual Hedge Price <u>75.00</u>

Higher Prices:

Cash price	Futures Price	Basis	Expected Hedge Price
	Sell <u>75.00</u>	<u>0</u> (estimated)	( <u>75.00</u> )
<u>82.50</u> (sell cash)	Buy <u>82.50</u>	<u>0</u> (actual)	
<u>82.50</u>	+ <u>-7.50</u>	=	Actual Hedge Price <u>75.00</u>

# HEDGING AND RISK SHIFTING SUMMARY

## Basis:

- ▶ The difference between your price for your hogs and the futures price on the date the hedge is lifted.
- ▶ Basis = Cash minus Futures
- ▶ Example: -\$2.00 (cash under futures) +2.00 (cash higher than futures)

## Estimated Basis:

- ▶ Based on historical cash/ futures price relationships
- ▶ Usually derived from 3-5 years of history
- ▶ Lean Hogs: Weighted average (about 54% lean)

## Actual Basis:

- ▶ Results in net hedge price when lifting hedge. Reflects how strong or weak the market is.
- ▶ Impacts your price

# OPTIONS AND FUTURES CONTRACT PRICE

June Lean Hog Futures = **76.00**

	Puts	Calls
In-the-money	77, 78, 79, 80	75, 74, 73, 72
At-the-money	76	76
Out-of-the-money	75, 74, 73, 72	77, 78, 79, 80

# JUNE LEAN HOG OPTIONS

LH/HE June Futures = 76.00

Put Strike	Puts Prem.	Call Strike	Calls Prem.
In the Money	78.00	3.00	74.00
	77.00	2.40	75.00
At Money	76.00	1.90	76.00
	75.00	1.50	77.00
Out of Money	74.00	1.00	78.00

3.40

2.85

2.00

1.75

1.25

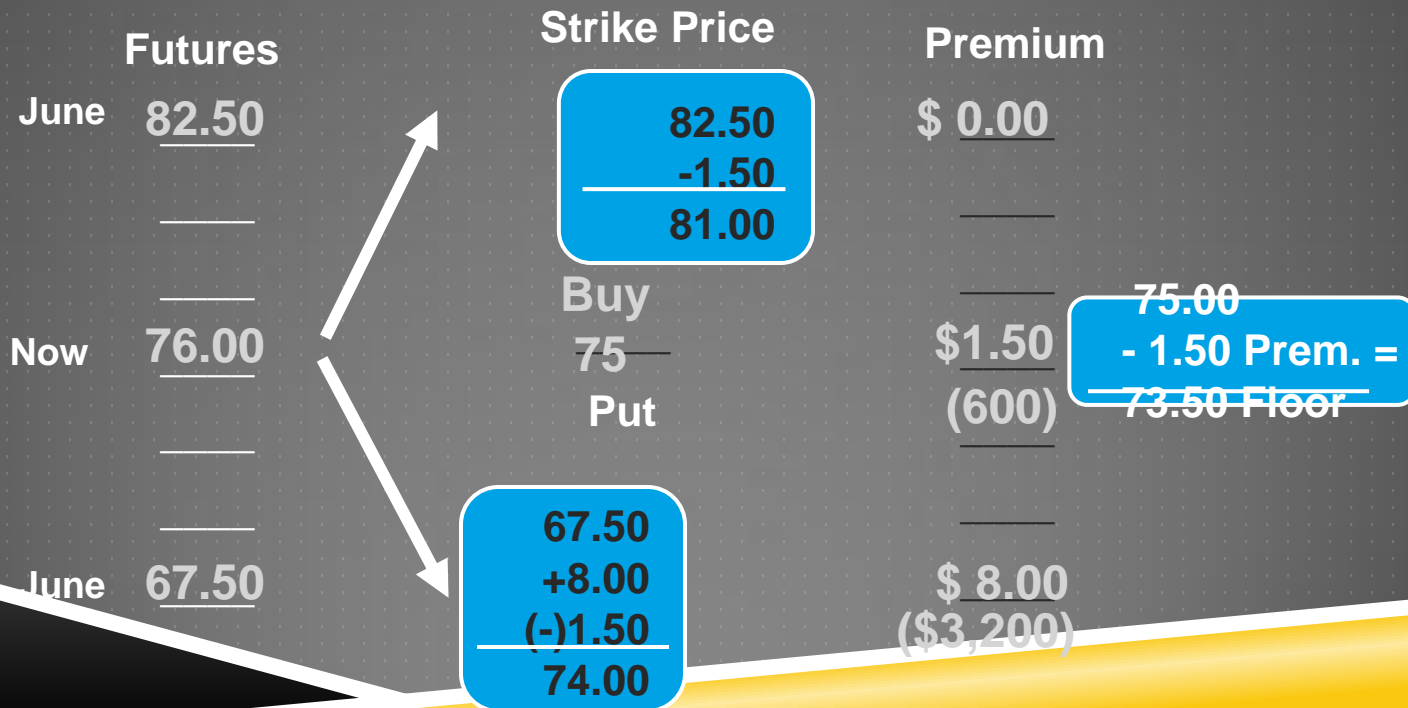
In the Money

At Money

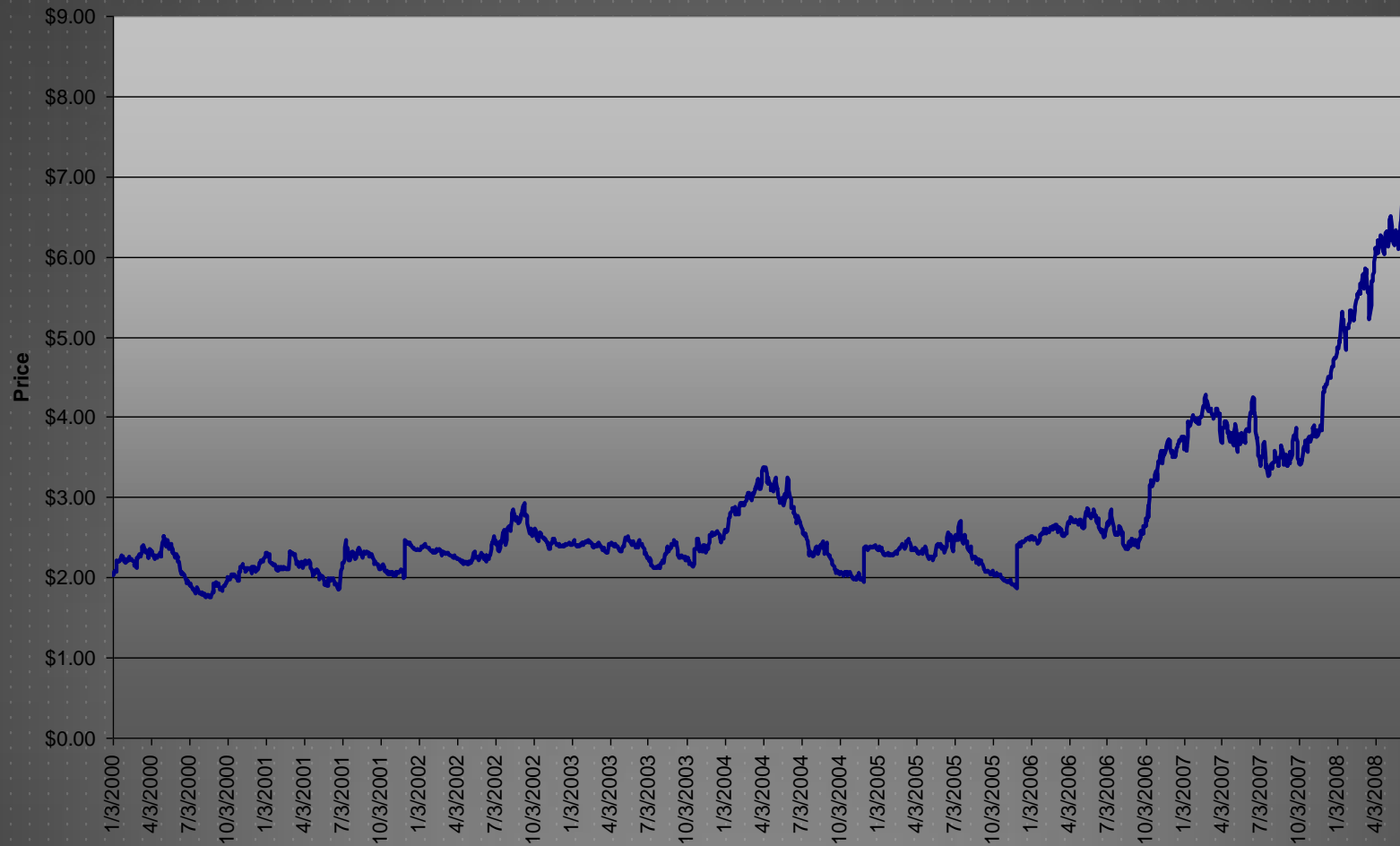
Out of Money

# PUT PREMIUM EXAMPLE

- ▶ Example: Buy a 75 Put when Futures are at \$ 76.00
- ▶ Increases in value as market falls
- ▶ Decreases in value to zero as market rises



# Corn Futures



# LONG HEDGING MANAGEMENT STRATEGIES

Buy Futures

Buy a Call

Roll Down to Futures

Sell a Put

Buy a Call & Sell a Put – Fence

# STEPS TO HEDGING SUMMARY

Know costs

Contract specifications

Basis

Performance bond requirements

Hedging costs

- ▶ Commission
- ▶ Interest

Knowledgeable

- ▶ Broker
- ▶ Lender

Marketing plan

# DEVELOPING A MARKETING PLAN

A decorative graphic at the bottom of the slide consists of a bright green trapezoidal shape pointing downwards, which is layered over a yellow trapezoidal shape pointing upwards. The two shapes meet at a white, jagged, zig-zagging line that separates them.

# MARKETING PLANS ARE CRITICAL TO SUCCESS

## A SEVEN-STEP MARKETING PLAN

1. Know your costs of production and determine your break-even levels
2. Utilize sound market information
3. Know your product
4. Set target price(s)
5. Evaluate pricing alternatives
  1. Forward contract
  2. Auction
  3. Futures/Options Hedging
6. Execute when target prices are hit.
7. Review results to determine what works best for your operation

# MARKETING ALTERNATIVES

Marketing Alternative	Advantages	Disadvantages
Cash sales	<ul style="list-style-type: none"> <li>• Easy to transact</li> <li>• Immediate payment</li> <li>• No set quantity</li> </ul>	<ul style="list-style-type: none"> <li>• Maximize risk</li> <li>• No price protection</li> <li>• Less flexible</li> </ul>
Forward contract	<ul style="list-style-type: none"> <li>• Easy to understand</li> <li>• Flexible quantity</li> <li>• Locked-in price</li> <li>• Minimize risk</li> </ul>	<ul style="list-style-type: none"> <li>• Must deliver in full</li> <li>• Opportunity loss if prices rise</li> </ul>
Futures contract	<ul style="list-style-type: none"> <li>• Easy to enter/exit</li> <li>• Minimize risk</li> <li>• Often better prices than forward contracts</li> </ul>	<ul style="list-style-type: none"> <li>• Opportunity loss if prices rise</li> <li>• Commission cost</li> <li>• Performance bond calls</li> <li>• Set quantities</li> </ul>
Options contract	<ul style="list-style-type: none"> <li>• Price protection</li> <li>• Minimize risk</li> <li>• Benefit if prices rise</li> <li>• Easy to enter/exit</li> </ul>	<ul style="list-style-type: none"> <li>• Premium cost</li> <li>• Set quantities</li> <li>• Commission cost</li> </ul>

# BOND FUNDAMENTALS

A decorative graphic at the bottom of the slide consists of a solid green trapezoidal shape pointing downwards, which is layered over a yellow trapezoidal shape also pointing downwards. Both shapes have a thin white border.

# PRICE-YIELD RELATIONSHIP

## ► Price of a Bond:

$$P = \sum_{t=1}^T \frac{C_t}{(1+y)^t} \quad (1.6)$$

$$P = f(y) \quad (1.7)$$

## Consols or Perpetual Bonds:

$$P = cF \left[ \frac{1}{(1+y)} + \frac{1}{(1+y)^2} + \frac{1}{(1+y)^3} + \dots \right] = \frac{c}{y} F \quad (1.8)$$

# PRICE-YIELD RELATIONSHIP

**FIGURE 1-1 Price-Yield Relationship**



# PRICE-YIELD RELATIONSHIP

- ▶ FRM is about assessing the effects of changes in risk factors, such as yields, on asset value
- ▶ What happens to the price of a bond if the yield changes from  $y_0$  to  $y_1$ ?
- ▶ Let's write  $y_1 = y_0 + \Delta y$
- ▶ Solutions:
  - ▶  $P_0 = f(y_0) \rightarrow P_1 = f(y_1)$

# TAYLOR EXPANSION

- ▶ If the change in the yield is not too large, the non linear relationship can be approximated by a Taylor Expansion

$$P_1 = P_0 + f'(y_0)\Delta y + \frac{1}{2}f''(y_0)(\Delta y)^2 + \dots \quad (1.9)$$

where

$f'(\cdot) = dP/dy$  is the 1<sup>st</sup> derivative

$f''(\cdot) = d^2P/dy^2$  is the 2<sup>nd</sup> derivative of the function  $f(\cdot)$  valued at the starting point

# DERIVATION OF DURATION MEASURE (I)

$$P = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \dots + \frac{C}{(1+y)^T} + \frac{F}{(1+y)^T}$$

$$\frac{dP}{dy} = \frac{-C}{(1+y)^2} + \frac{(-2)C}{(1+y)^3} + \dots + \frac{(-T)C}{(1+y)^{T+1}} + \frac{(-T)F}{(1+y)^{T+1}}$$

$$\frac{dP}{dy} = -\frac{1}{1+y} \left[ \frac{C}{1+y} + \dots + \frac{TC}{(1+y)^T} + \frac{TF}{(1+y)^T} \right]$$

$$\frac{dP}{dy} \frac{1}{P} = -\frac{1}{1+y} \left[ \frac{C}{1+y} + \dots + \frac{TC}{(1+y)^T} + \frac{TF}{(1+y)^T} \right] \frac{1}{P}$$

## DERIVATION OF DURATION MEASURE (2)

$$D = \frac{\frac{C}{1+y} + \frac{2C}{(1+y)^2} + \dots + \frac{TC}{(1+y)^T} + \frac{TF}{(1+y)^T}}{P}$$

$$D = \frac{\sum_{t=1}^T \frac{tC}{(1+y)^t} + \frac{TF}{(1+y)^T}}{P}$$

$$\frac{dP}{dy} = -\frac{1}{1+y} DP$$

# BOND PRICE DERIVATIVES

$$f'(y_0) = \frac{dP}{dy} = -D^* \times P_0 \quad (1.12)$$

$D^*$  is modified duration:  $D^* = D/(1 + y)$

Dollar duration:

$$DD = D^* \times P_0 \quad (1.13)$$

Risk in bonds can be measured as the dollar value of a basis point (DVBP)

$$DVBP = [D^* \times P_0] \times 0.0001 \quad (1.14)$$

# DURATION: ZERO COUPON BOND

$$\frac{dP}{dy} = \frac{d}{dy} \left[ \frac{F}{(1+y)^T} \right] = (-T) \frac{F}{(1+y)^{T+1}} = -\frac{T}{(1+y)} P \quad (1.16)$$

$$D^* = T/(1+y) \quad \rightarrow \quad D = T$$

Duration is expressed in periods of time: T

With annual compounding Duration is in years, with semiannual compounding duration is in semesters.

# SECOND DERIVATIVE: CONVEXITY

- ▶ Duration is the first derivative of the price-yield function. Convexity is the second derivative of the price-yield function. Convexity is the rate of change in modified duration as yields shift. Convexity is expressed in periods squared

## Dollar Convexity

$$f''(y_0) = \frac{d^2 P}{dy^2} = C \times P_0 \quad (1.15)$$

## Zero Coupon Bond

$$\frac{d^2 P}{dy^2} = -(T+1)(-T) \frac{F}{(1+y)^{T+2}} = \frac{(T+1)T}{(1+y)^2} \times P \quad (1.18)$$

$$C = (T+1)T/(1+y)^2$$

# TAYLOR EXPANSION

$$\Delta P = -[D^* \times P](\Delta y) + \frac{1}{2}[C \times P](\Delta y)^2 + \dots \quad (1.19)$$

Example: 10-year zero coupon bond,  $y = 6\%$

$$P = 100/(1 + 0.03)^{20} = \$55.37$$

$D = 20$  semesters (10 years), ( $D = T$  for zero-coupon bond)

$$D^* = T/(1 + y) = 20/(1 + 0.03) = 19.42 \text{ (9.71 years)}$$

$$C = 21 \times 20/(1 + 0.03)^2 = 395.89 \text{ semesters squared (98.97 years squared)}$$

$$DD = D^* \times P = 9.71 \times \$55.37 = \$537.55$$

Yield is moving from 6% to 7%

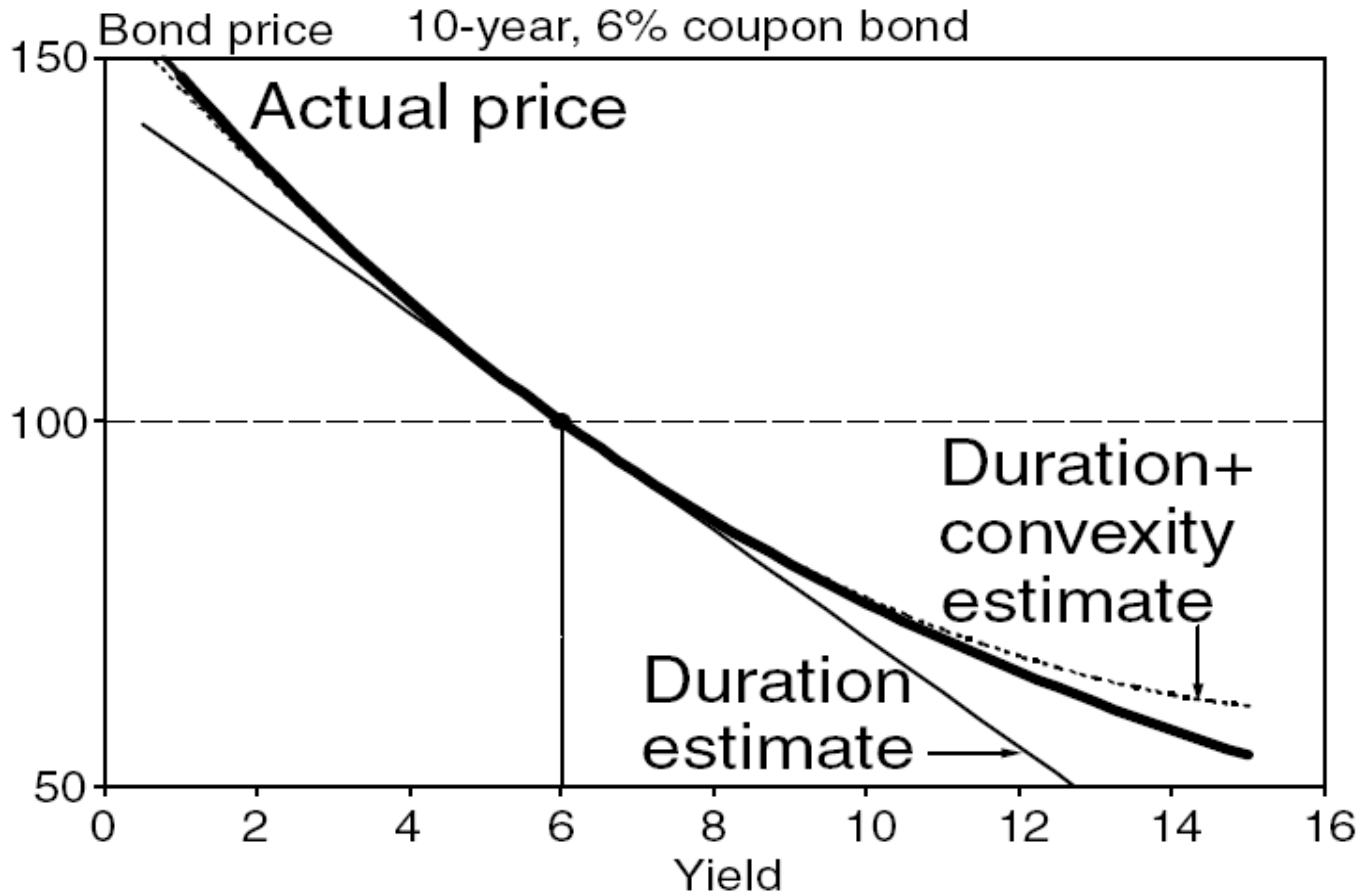
$$\Delta P = -[9.71 \times 55.37](0.01) + 0.5[98.97 \times 55.37](0.01)^2 = -5.375 + 0.274$$

$$P_1 = P_0 + \Delta P = 55.37 - 5.375 + 0.274 = \$50.266$$

The exact value  $P_1 = f(y_1) = \$50.257$

# DURATION AND CONVEXITY

**FIGURE 1-2 Price Approximation**



# DURATION AND CONVEXITY

- ▶ Dollar Duration measures the (negative) slope of the tangent to the price-yield curve at the starting point
- ▶ Convexity is always positive for regular coupon-paying bonds. Greater convexity is beneficial for both falling and raising yields → bond price moves less!

# NUMERICAL METHODS

- ▶ Duration and convexity cannot be computed directly for some bonds when the cash flows are uncertain. We may estimate them as follows
- ▶ Effective Duration

$$D^E = \frac{[P_- - P_+]}{(2P_0\Delta y)} = \frac{P(y_0 - \Delta y) - P(y_0 + \Delta y)}{(2\Delta y)P_0} \quad (1.20)$$

- ▶ Effective Convexity

$$C^E = [D_- - D_+]/\Delta y = \left[ \frac{P(y_0 - \Delta y) - P_0}{(P_0\Delta y)} - \frac{P_0 - P(y_0 + \Delta y)}{(P_0\Delta y)} \right] / \Delta y \quad (1.21)$$

where  $P_+ = P(y_0 + \Delta y) \rightarrow$  up-move scenario

$P_- = P(y_0 - \Delta y) \rightarrow$  down-move scenario

# INTERPRETING DURATION

- ▶ Example: 10-year bond,  $y = 6\%$ ,  $D = 7.80$  years

This coupon paying bond is equivalent to a zero-coupon bond maturing in 7.8 years

- ▶ A measure of the average life of an investment (controversial).
- ▶ For a zero coupon bond the duration is the period of time remaining until the bond matures. For a coupon bond the duration is less than the term to maturity since interest is being paid back each six months.
- ▶ Macaulay argued that the best descriptive measure of a bond's average life should consider all of the bond's cash flows as well as the time value of money.
- ▶ Duration is defined as the weighted average maturity of a bond's cash flows where the present values of the cash flows serve as the weights.

# INTERPRETING DURATION

$$\frac{dP}{dy} = \sum_{t=1}^T \frac{-tC_t}{(1+y)^{t+1}} = - \left[ \sum_{t=1}^T \frac{tC_t}{(1+y)^t} \right] / P \times \frac{P}{(1+y)} = - \frac{D}{(1+y)} P \quad (1.23)$$

$$D = \sum_{t=1}^T \frac{tC_t}{(1+y)^t} / P \quad (1.24)$$

$$D = \sum_{t=1}^T t \frac{C_t / (1+y)^t}{\sum C_t / (1+y)^t} = \sum_{t=1}^T t \times w_t \quad (1.25)$$

# INTERPRETING DURATION: CONSOLS

►  $P = (c/y)F$

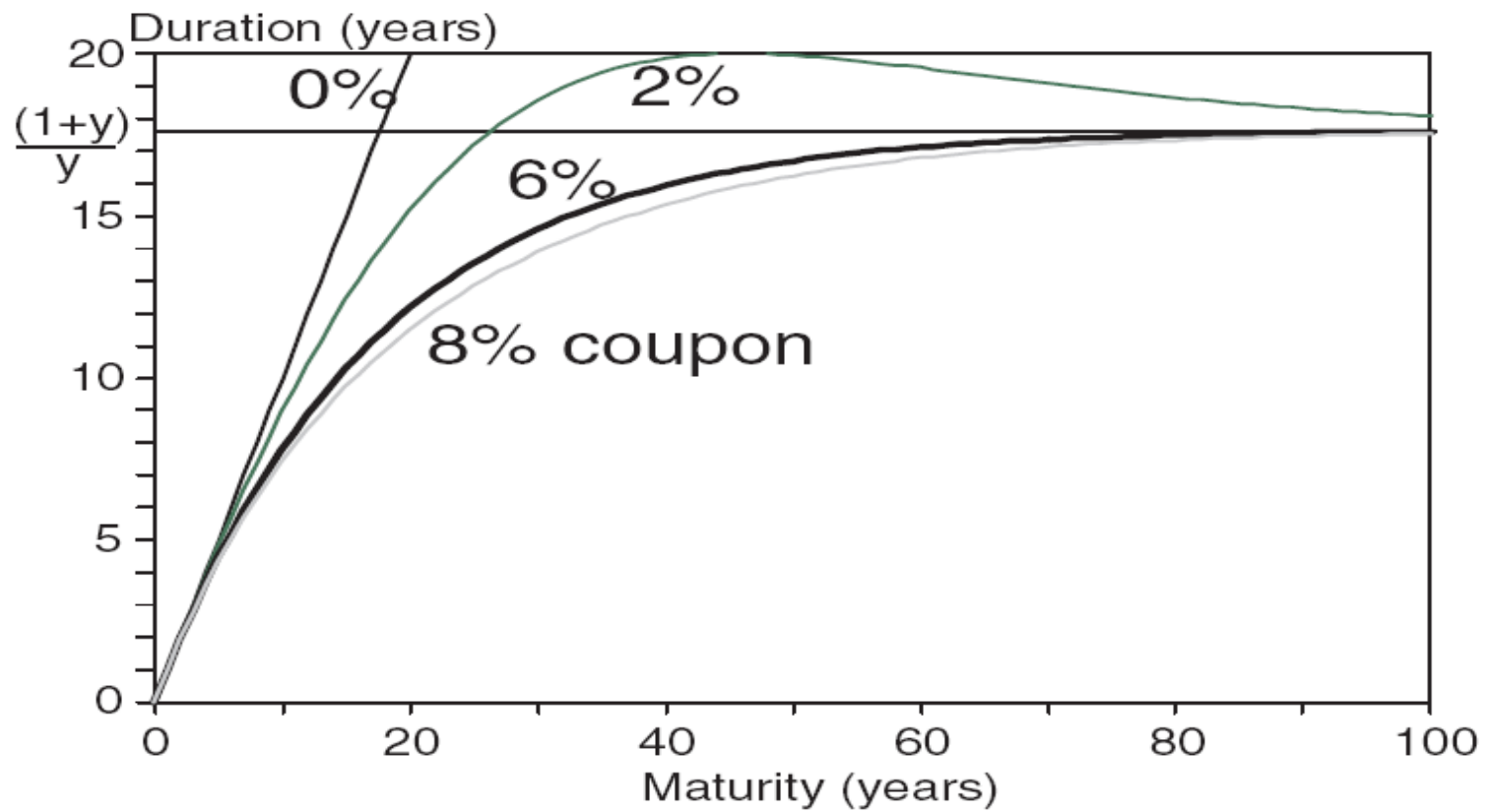
$$\frac{dP}{dy} = cF \frac{(-1)}{y^2} = (-1) \frac{1}{y} \left[ \frac{c}{y} F \right] = (-1) \frac{1}{y} P = -\frac{D_C}{(1+y)} P \quad (1.26)$$

$$D_C = \frac{(1+y)}{y} \quad (1.27)$$

- Duration of a consol is finite even if its maturity is infinite and it does not depend on the coupon.
- The duration of a long-term bond can be approximated by an upper bound equal to the duration of a consol with the same yield

# DURATION AND MATURITY: 6% YIELD ENVIRONMENT

**FIGURE 1-7 Duration and Maturity**



# INTERPRETING CONVEXITY

$$\frac{d^2 P}{dy^2} = \sum_{t=1}^T \frac{t(t+1)C_t}{(1+y)^{t+2}} = \left[ \sum_{t=1}^T \frac{t(t+1)C_t}{(1+y)^{t+2}} / P \right] \times P \quad (1.28)$$

$$C = \sum_{t=1}^T \frac{t(t+1)C_t}{(1+y)^{t+2}} / P \quad (1.29)$$

$$C = \sum_{t=1}^T \frac{t(t+1)}{(1+y)^2} \times \frac{C_t / (1+y)^t}{\sum C_t / (1+y)^t} = \sum_{t=1}^T \frac{t(t+1)}{(1+y)^2} \times w_t \quad (1.30)$$

- ▶ Convexity: weighted average of the square of time
- ▶ Convexity is much greater for long-term bonds because they have cash-flows associated with large values of  $t$
- ▶ Zero-coupon bond has greater convexity because there is only one cash-flow at maturity (convexity is roughly the square of maturity)

**TABLE 1-2 Computing Duration and Convexity**

Period (half-year) $t$	Payment $C_t$	Yield (%) (6 mo)	$PV$ of Payment $C_t/(1+y)^t$	Duration Term $tPV_t$	Convexity Term] $t(t+1)PV_t$ $\times(1/(1+y)^2)$
1	3	3.00	2.913	2.913	5.491
2	3	3.00	2.828	5.656	15.993
3	3	3.00	2.745	8.236	31.054
4	103	3.00	91.514	366.057	1725.218
Sum:			100.00	382.861	1777.755
(half-years)				3.83	17.78
(years)				1.91	
Modified duration				1.86	
Convexity					4.44

# PORTFOLIO DURATION AND CONVEXITY

## ► Portfolio duration

$$D_p^+ = \sum_{i=1}^N D_i^+ w_i \quad (1.33)$$

## ► Portfolio convexity

$$C_p = \sum_{i=1}^N C_i w_i \quad (1.34)$$