

Johns Hopkins University  
Carey Business School  
Financial Risk Management

Fall 2009

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**ASSIGNMENT – DUE ON Thursday November 26, 2009**

Show your work. Do NOT just apply formulae. The focus of the homework is not on right or wrong numerical answers. Instead, the emphasis is on your ability to demonstrate your solid understanding of the concepts, and your ability to apply the concepts to real data. Numerical accuracy is less important than the reasoning and narrative. In general the answers must be complete and demonstrate a clear and full understanding of the concepts or parameters being described.

Your work will be evaluated based on overall quality and presentation.

**IMPORTANT:** This is an individual assignment. University regulation on plagiarism will be applied.

Submit a written report and a CD containing your calculations.

Late submissions will not be graded.

**Question 1** (45 points)

Download the S&P 500 Index (symbol ^GSPC) from November 1st 2008 to October 30th 2009. Assume you have a portfolio worth \$100 M (at current market values) invested uniquely in the stock index.

- Compute daily returns on the stock:  $R_t = \ln(P_t / P_{t-1}) \times 100$ . (5 points)
- Compute the VaR using the historical distribution and normal distribution. Comment on your results. (10 points)
- Assume that a trading day runs from 9:30 am EST to 4:00 pm EST. Further assume that the price process is observed every 5 minutes. This implies that there are a total of 79 observations per day. The price process follows a geometric Brownian motion

$$\Delta S = \mu S \Delta t + \sigma S \Delta z = \mu S \Delta t + \sigma S \sqrt{\Delta t} \varepsilon$$

where  $\mu = 7\%$  per annum,  $\sigma = 36\%$  per annum,  $S_0 = \$1066.95$  (this is the initial price). Simulate a trading week (5 trading days). Each trading day is composed by 79 price observations. The final observation for each trading day is the close price. [Hint: be careful about  $\Delta t$ .] (10 points)

- From the simulated close prices in c) compute rate of returns and then VaR from [historical distribution](#). Comment on your results. (5 points)
- From the simulated close prices in c) compute rate of returns and then VaR assuming that the underline risk factor follows a [Normal distribution](#). Comment on your results. (5 points)
- Based on the same assumptions ( $\mu$ ,  $\sigma$ , and  $S_0$ ) in question c), if the price process is observed every 1 second, please simulate transaction prices next trading day. [Hint: Each trading day is composed by 23401 price observations]. What are the main assumptions in your answers? (10 points)

**Question 2** (25 points)

The current stock price is \$52, the strike price is \$50, the risk free rate of interest is 10% per annum, the volatility of the underline stock is 30% per annum, and the time to maturity is three months.

- a) Compute the Greek letters for this option assuming that the famous Black-Scholes option pricing model holds. Comment on your results. (15 points)

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- b) Why we need these Greek letters? What's the relationship among these Greek letters? (10 points)

**Question 3 (10 points)**

Assume that the stock price,  $S$ , evolves according to the following process

$$dS = \mu S dt + \sigma S \varepsilon \sqrt{dt}$$

- a) Consider the function  $G = S \left( \exp^{r(T-t)} - 1 \right)$ . Compute the process for  $G$ . (5 points)
- b) An option on the stock  $S$ , can be written as a function of  $S$  and time:  $f(S, t)$ . Compute the process for  $f$ . Comment on your results. (5 points)

**Question 4 (15 points)**

Consider the following GARCH model

$$r_t = \mu + \varepsilon_t$$

$$\varepsilon_t = h_t^{1/2} e_t$$

$$e_t \sim N(0,1)$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}$$

- a) The estimation results are as follows. Please solve the parameters in the table and comment your result (6 points)

|            | Parameter | Standard Error | t-value |
|------------|-----------|----------------|---------|
| $\mu$      | 0.06      | a              | 3       |
| $\alpha_0$ | 0.02      | b              | 2       |
| $\alpha_1$ | 0.04      | c              | 4       |
| $\beta$    | 0.94      | 0.02           | d       |

- b) Based on the following data, please solve the following parameters. (e, f, and g). based on GARCH Model and Risk Metric model with  $\lambda = 0.97$  (6 points)

| Rate of Return | Volatility |
|----------------|------------|
| -0.489869962   | 0.354148   |
| 1.629695715    | e          |
| 0.366603354    | f          |
| 0.001570512    | g          |

- c) Compare and comment the results in b) and c). (3 points)

**Question 5** (5 points)

The following data refers to call option prices traded today on the same asset. The asset does not pay any income. Using the standard Black-Scholes formula derives the implied volatilities of these options. (5 points)

| Spot Price | Strike Price | Maturity (days) | Interest Rate | Option Price |
|------------|--------------|-----------------|---------------|--------------|
| \$100      | \$90         | 90              | 2.00%         | \$13.79      |
| \$100      | \$95         | 90              | 2.00%         | \$9.74       |
| \$100      | \$100        | 90              | 2.00%         | \$6.37       |
| \$100      | \$105        | 90              | 2.00%         | \$5.02       |
| \$100      | \$110        | 90              | 2.00%         | \$4.16       |