

HEDGING LINEAR RISK

A decorative graphic at the bottom of the slide consists of a solid green trapezoidal shape pointing downwards, which is partially overlapped by a yellow trapezoidal shape pointing upwards. Both shapes have a white border.

HEDGING

- ▶ Risk that has been measured can be managed
- ▶ Hedging: taking positions that lower the risk profile of the portfolio
Hedging → developed in the futures markets, where farmers use financial instruments to hedge the price risk of their products
- ▶ Hedging linear risk: futures → linearly related to the underlying risk factor
- ▶ Objective: find the optimal position that minimize variance (standard deviation) or VaR
- ▶ Portfolio consists of two positions: asset to be hedge and hedging instrument

HEDGING: BASIC IDEA

- ▶ Short Hedge: A company that knows (time 0) that it is due to sell an asset at a particular time in the future (time 1) can hedge by taking a short futures position
 - ▶ $S_1 \downarrow \implies S_1 < S_0 \implies$ loss in the underlying asset $S_0 - S_1$, profit in the short future position $F_{01} > S_1$
 - ▶ $S_1 \uparrow \implies S_1 > S_0 \implies$ profit in the underlying asset $S_1 - S_0$, loss in the short future position $F_{01} < S_1$
- ▶ Long Hedge: A company that knows (time 0) that it is due to buy an asset at a particular time in the future (time 1) can hedge by taking a long futures position
 - ▶ $S_1 \downarrow \implies S_1 < S_0 \implies$ profit in the underlying asset $S_0 - S_1$, loss in the long future position $F_{01} > S_1$
 - ▶ $S_1 \uparrow \implies S_1 > S_0 \implies$ loss in the underlying asset $S_1 - S_0$, profit in the long future position $F_{01} < S_1$

HEDGING:TWO STRATEGIES

- ▶ **Static hedging** → putting on, and leaving, a position until the hedging horizon
- ▶ **Dynamic hedging** → continuously rebalancing the portfolio to the horizon. This can create a risk profile similar to positions in options
- ▶ **Important**: futures hedging does not necessarily improve the overall financial outcome (roughly speaking, the probability that a future hedge will make the outcome worse is 0.5)
- ▶ What the futures hedge does do is reduce risk by making the outcome more certain

HEDGING: EXAMPLE

- ▶ A U.S. exporter will receive a payment of 125M Japanese yen in 7 months. This is a cash position (or anticipated inventory)
- ▶ The perfect hedge would be to enter a seven-month forward contract OTC
Unfortunately, there is nobody willing to meet the needs of the U.S. Exporter
- ▶ The exporter decides to turn to an exchange-traded futures contract, which can be bought or sold easily
- ▶ The Chicago Mercantile Exchange (CME) lists yen contracts with a face amount of ¥12,500,000 that expire in nine months. The exporter places an order to sell 10 contracts, with the intention of reversing the position in seven months, when the contract will still have two months to maturity
- ▶ Because the amount sold is the same as the underlying → **unitary hedge**
- ▶ Suppose that the yen depreciates sharply, or that the dollar goes up from ¥125 to ¥150. Loss in the cash position $¥125M(0.006667-0.00800) = - \$166,667$
- ▶ Gain on the futures, which is $(-10)¥125M(0.006667-0.00806) = \$168,621$

BASIS RISK

- ▶ P&L of unhedged position

- ▶ P&L of hedged position $Q[S_2 - S_1]$ (12.1)

$$Q[(S_2 - S_1) - (F_2 - F_1)] = Q[(S_2 - F_2) - (S_1 - F_1)] = Q[b_2 - b_1] \quad (12.2)$$

- ▶ Problems:

- ▶ The asset whose price is to be hedge may not be exactly the same as the asset underlying the futures contract
- ▶ The hedger may be uncertain as to the exact date when the asset will be bought or sold
- ▶ The hedge may require the futures contract to be closed out well before its expiration date

BASIS RISK

- ▶ For investments assets - currencies, stock indices, gold and silver - the basis risk tends to be small. This is because there is a well-defined relationship between the future price and the spot price
 - ▶ $F_{0T} = e^{rT}S_0$
 - ▶ the basis risk arises mainly from uncertainty about the risk-free interest rate
- ▶ For commodities - crude oil, corn, copper, etc. - supply and demands effects can lead to large variation in the basis ==> higher basis risk
- ▶ **Cross hedging:** using a futures contract on a totally different asset or commodity than the cash position

S_2^* is the asset underlying the futures contract

$$(S_2^* - F_2) - (S_1 - F_1) + (S_2 - S_2^*)$$

BASIS RISK

- ▶ A key factor affecting basis risk is the choice of the futures contract to be used for hedging. This choice has two components:
 - ▶ The choice of the asset underlying the future contracts
 - ▶ case by case analysis
 - ▶ The choice of the delivery month
 - ▶ No the same month (high volatility during the delivery month)
 - ▶ In general, basis risk increases as the time difference between the hedge expiration and the delivery month increases
 - ▶ Good rule: to choose a delivery month that is as close as possible to, but later than the expiration of the hedge

OPTIMAL HEDGE RATIO

- ▶ The hedge ratio is the ratio of the size of the position taken in futures contracts to the size of the exposure (up to now we have assumed a hedge ratio = 1)
- ▶ Definitions:
 - ▶ ΔS : change in the spot price S , ΔF : change in the futures price F
 - ▶ $\sigma_{\Delta S}$: standard deviation of ΔS , $\sigma_{\Delta F}$: standard deviation of ΔF
 - ▶ $\sigma_{\Delta F, \Delta F}$: covariance between ΔS and ΔF
 - ▶ $\rho_{\Delta S, \Delta F}$: correlation between ΔS and ΔF
 - ▶ N : number of futures contracts to buy/sell to hedge (hedge ratio)
- ▶ Short hedge: $\Delta S - N\Delta F \rightarrow \Delta V = \Delta S - N\Delta F$
- ▶ Long Hedge: $N\Delta F - \Delta S \rightarrow \Delta V = N\Delta F - \Delta S$

OPTIMAL HEDGE RATIO

- ▶ The variance of the change in portfolio value is equal to

$$\sigma_{\Delta V}^2 = \sigma_{\Delta S}^2 + N^2 \sigma_{\Delta F}^2 - 2N \sigma_{\Delta V, \Delta F}$$

- ▶ To minimize the variance, the only choice variable is $N \rightarrow$ we compute the 1st derivative with respect to N and set it equal to zero

$$\frac{\partial \sigma_{\Delta V}^2}{\partial N} = 2N \sigma_{\Delta F}^2 - 2 \sigma_{\Delta V, \Delta F}$$

Noting that

$$\partial^2 \sigma_{\Delta V}^2 / \partial N^2 > 0$$

$$N^* = \frac{\sigma_{\Delta S, \Delta F}}{\sigma_{\Delta F}^2} = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}}$$

OPTIMAL HEDGE RATIO

- ▶ Example: An airline company knows that it will buy 1 million gallons of jet fuel in 3 months. The st. dev. of the change in price of jet fuel is calculated as 0.032. The company choose to hedge by buying futures contracts on heating oil. St. dev. of heating oil is equal to 0.04 and $\rho=0.8 \implies h = 0.8(0.032/0.04) = 0.64$.
 - ▶ One heating oil futures contract is on 42,000 gallons. The company should therefore buy $0.64(1,000,000/42,000) = 15.2$.

OPTIMAL HEDGE RATIO

- ▶ It is sometimes easier to deal with unit prices and to express volatilities in terms of *rates of changes in unit prices* → quantities Q and unit prices s , we have notional amount of cash position, $S = Qs$ and the notional amount of one futures contract $F = Q_f f$
- ▶ We can then write

$$\sigma_{\Delta S} = Q\sigma(\Delta s) = Qs\sigma(\Delta s/s)$$

$$\sigma_{\Delta F} = Q_f\sigma(\Delta f) = Q_f f\sigma(\Delta f/f)$$

$$\sigma_{\Delta S, \Delta F} = \rho_{sf}[Qs\sigma(\Delta s/s)][Q_f f\sigma(\Delta f/f)]$$

$$N^* = \rho_{SF} \frac{Qs\sigma(\Delta s/s)}{Q_f f\sigma(\Delta f/f)} = \rho_{SF} \frac{\sigma(\Delta s/s)}{\sigma(\Delta f/f)} \frac{Qs}{Q_f f} = \beta_{sf} \frac{Qs}{Q_f f}$$

where β_{SF} is the coefficient in the regression of $\frac{\Delta s}{s}$ on $\frac{\Delta f}{f}$

OPTIMAL HEDGE RATIO

- ▶ The optimal amount N^* can be derived from the slope coefficient of a regression of $\Delta s/s$ on $\Delta f/f$:

$$\frac{\Delta s}{s} = \alpha + \beta_{sf} \frac{\Delta f}{f} + \epsilon \quad (12.8)$$

- ▶ Let's compute the minimum variance by plugging the variance of total profit function

$$\beta_{sf} = \frac{\sigma_{sf}}{\sigma_f^2} = \rho_{sf} \frac{\sigma_s}{\sigma_f}$$

$$\sigma_V^{*2} = \sigma_S^2 + \left(\frac{\sigma_{SF}}{\sigma_F^2} \right)^2 \sigma_F^2 + 2 \left(\frac{-\sigma_{SF}}{\sigma_F^2} \right) \sigma_{SF} = \sigma_S^2 + \frac{\sigma_{SF}^2}{\sigma_F^2} + 2 \frac{-\sigma_{SF}^2}{\sigma_F^2} = \sigma_S^2 - \frac{\sigma_{SF}^2}{\sigma_F^2} \quad (12.10)$$

$$R^2 = \frac{\sigma_S^2 - \sigma_V^{*2}}{\sigma_S^2} \quad (12.11)$$

OPTIMAL HEDGE RATIO

- ▶ This regression gives us the effectiveness of the hedge, which is measured by the proportion of variance eliminated

$$\sigma_V^* = \sigma_S \sqrt{(1 - R^2)}$$

- ▶ If $R^2=1$, then the resulting portfolio has zero risk

OPTIMAL HEDGE RATIO: EXAMPLE

- ▶ An airline knows that it will need to purchase 10,000 metric tons of jet fuel in three months. It wants some protection against an upturn in prices using futures contracts.
- ▶ The company can hedge using heating oil futures contracts traded on NYMEX. The notional for one contract is 42,000 gallons.
- ▶ There is no futures contract on jet fuel, the risk manager wants to check if heating oil could provide an efficient hedge. The current price of jet fuel is \$277/metric ton. The futures price of heating oil is \$0.6903/gallon.
- ▶ The standard deviation of the rate of change in jet fuel prices over three months is 21.17%, that of futures is 18.59%, and the correlation is 0.8243
- ▶ **Compute**
 - a) The notional and the standard deviation of the unhedged fuel cost in dollars
 - b) The optimal number of futures contract to buy/sell, rounded to the closest integer
 - c) The standard deviation of the hedged fuel cost in dollars

OPTIMAL HEDGE RATIO: EXAMPLE

- ▶ The position notional is $Q_s = \$2,770,000$. The standard deviation in dollars is

$$\sigma(\Delta s/s)sQ = 0.2117 \times \$277 \times 10,000 = \$586,409$$

that of one futures contract is

$$\sigma(\Delta f/f)fQ_f = 0.1859 \times \$0.6903 \times 42,000 = \$5,389.72$$

and $fQ_f = \$0.6903 \times 42,000 = \$28,992.60$

OPTIMAL HEDGE RATIO: EXAMPLE

- ▶ The cash position corresponds to a payment, or liability. Hence, the company will have to *buy* futures as protection
- ▶ $\beta_{sf} = 0.8243(0.2117 / 0.1859) = 0.9387$
- ▶ $\sigma_{sf} = 0.8243 \times 0.2117 \times 0.1859 = 0.03244$
- ▶ $\sigma_{SF} = 0.03244 \times \$2,770,000 \times \$28,993 = 2,605,268,452$

$$N^* = \beta_{sf} \frac{Q \times s}{Q_f \times f} = 0.9387 \frac{10,000 \times \$277}{42,000 \times \$0.69} = 89.7$$

OPTIMAL HEDGE RATIO: EXAMPLE

- ▶ To find the risk of the hedged position, we use

$$\sigma_V^{*2} = \sigma_S^2 - (\sigma_{SF}^2 / \sigma_F^2)$$

$$\sigma_S^2 = (\$586,409)^2 = 343,875,515.281$$

$$\sigma_{SF}^2 / \sigma_F^2 = -(2,605,268,452 / 5,390^2) = -233,653,264,867$$

$$\sigma_V^{*2} = 110,222,250,414$$

$$\sigma_V^* = \$331,997$$

- ▶ The hedge has reduced risk from \$586,409 to \$331,997
- ▶ $R^2 \rightarrow$ effectiveness of the hedge = 67.95%

HEDGING: BE CAREFUL

Futures hedging can be successful in reducing market risk

BUT

it can create other risks → Futures contracts are marked to market daily → Hence they can involve large cash inflows or outflows → liquidity problems, especially when they are not offset by cash inflows from the underlying position

OPTIMAL HEDGE RATIO FOR BONDS

► We know that: $\Delta P = (-D^*P) \Delta y$

► For the cash and futures positions:

$$\Delta S = (-D_S^* S) \Delta y$$

$$\Delta F = (-D_F^* F) \Delta y$$

$$\sigma_S^2 = (D_S^* S)^2 \sigma^2(\Delta y) \quad \sigma_F^2 = (D_F^* F)^2 \sigma^2(\Delta y) \quad \sigma_{SF} = (D_F^* F)(D_S^* S) \sigma^2(\Delta y)$$

$$N^* = -\frac{\sigma_{SF}}{\sigma_F^2} = -\frac{(D_F^* F)(D_S^* S)}{(D_F^* F)^2} = -\frac{(D_S^* S)}{(D_F^* F)} \quad (12.14)$$

OPTIMAL HEDGE RATIO FOR BONDS: EXAMPLE

A portfolio manager holds a bond portfolio worth \$10 million with a modified duration of 6.8 years, to be hedged for three months. The current futures price is 93-02, with a notional of \$100,000. We assume that its duration can be measured by that of the cheapest-to-deliver, which is 9.2 years

Compute

- a) The notional of the futures contract
- b) The number of contracts to buy/sell for optimal protection

OPTIMAL HEDGE RATIO FOR BONDS: EXAMPLE

- ▶ The notional is $[93 + (2/32)]/100 \times \$100,000 = \$93,062.5$
- ▶ The optimal number to sell is

$$N^* = -\frac{(D_S^* S)}{(D_F^* F)} = -\frac{6.8 \times \$10,000,000}{9.2 \times \$93,062.5} = -79.4$$

On February 2, a corporate treasurer wants to hedge a July 17 issue of \$5 million of commercial paper with a maturity of 180 days, leading to anticipated proceeds of \$4.52 million. The September Eurodollar futures trades at 92 and has a notional amount of \$1 million.

Compute

- a) The current dollar value of the futures contract
- b) The number of contracts to buy/sell for optimal protection

Answer

a) The current dollar price is given by $\$10,000[100 - 0.25(100 - 92)] = \$980,000$. Note that the duration of the futures is always three months (90 days), since the contract refers to three-month LIBOR.

b) If rates increase, the cost of borrowing will be higher. We need to offset this by a gain, or a short position in the futures. The optimal number is, from Equation (12.14),

$$N^* = -\frac{(D_S^* S)}{(D_F^* F)} = -\frac{180 \times \$4,520,000}{90 \times \$980,000} = -9.2$$

or 9 contracts after rounding. Note that the DVBP of the futures is about $0.25 \times \$1,000,000 \times 0.01\% = \25 .

BETA HEDGING

Beta, or **systematic risk**, can be viewed as a measure of the exposure of the rate of return on a portfolio i to movements in the “market” m

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it} \quad (12.16)$$

We can also write this as

$$(\Delta S/S) \approx \beta(\Delta M/M) \quad (12.17)$$

Assume that we have at our disposal a stock index futures contract, which has a beta of unity $\rightarrow (\Delta F / F) = 1(\Delta M / M)$

BETA HEDGING

- Therefore, we can write

$$\Delta V = \Delta S - N\Delta F = (\beta S) (\Delta M / M) - NF (\Delta M / M)$$

$$\Delta V = [\beta S - NF] (\Delta M / M)$$

$$\Delta V = 0 \quad \text{if} \quad N^* = \beta(S / F)$$

Key concept:

The optimal hedge with stock index futures is given by the the beta of the cash position times its value divided by the notional of the futures contract.

The quality of the hedge will depend on the size of the residual risk in the market model. For large portfolios, the approximation may be good. In contrast, hedging an individual stock with stock index futures may give poor results

BETA HEDGING: EXAMPLE

A portfolio manager holds a stock portfolio worth \$10 million with a beta of 1.5 relative to the S&P 500. The current futures price is 1,400, with a multiplier of \$250.

Compute

- The notional of the futures contract
- The number of contracts to sell short for optimal protection

Answer

- The notional amount of the futures contract is $\$250 \times 1,400 = \$350,000$.
- The optimal number of contract to short is, from Equation (12.18),

$$N^* = -\frac{\beta S}{F} = -\frac{1.5 \times \$10,000,000}{1 \times \$350,000} = -42.9$$

or 43 contracts after rounding.

REASONS FOR HEDGING AN EQUITY PORTFOLIO

- ▶ Desire to be out of the market for a short period of time. (Hedging may be cheaper than selling the portfolio and buying it back.)
- ▶ Desire to hedge systematic risk (Appropriate when you feel that you have picked stocks that will outperform the market.)

HEDGING: PROS AND CONS

- ▶ Pros:

Companies should focus on the main business they are in and take steps to minimize risks arising from interest rates, exchange rates, and other market variables

- ▶ Cons:

Shareholders are usually well diversified and can make their own hedging decisions

It may increase risk to hedge when competitors do not

Explaining a situation where there is a loss on the hedge and a gain on the underlying can be difficult

Open Interest 10-year Treasury Notes Futures

