



Financial Econometrics

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- “Financial econometrics has emerged as one of the vibrant areas ... featuring an explosion of theoretical and applied work. ... The two have gone together, with the development of sophisticated techniques being driven by the special features seen in financial data.” (Pagan, 1996, page 15).
- The goal is to model financial variables in order to:
 - understand financial markets ==> backward-looking interpretation
 - forecast future behaviours ==> forward-looking interpretation

Stylised Facts



- 1 The unconditional distribution of returns on financial assets is non-normal (leptokurtosis)
 - 2 The variance is not constant but changes over time (heteroskedasticity)
 - 3 Large (small) changes in the conditional variance are followed by large (small) changes (volatility clustering)
- See Mandelbrot (1963a, 1963b), Fama (1965)

Table 1**Summary Statistics of Daily Stock Returns**

	US	UK	Germany	France	Italy
Mean (%)	0.043 (F)	0.046 (F)	0.031 (F)	0.044 (F)	0.053 (F)
	0.045 (A)	0.069 (B)	0.042 (B)	0.065 (B)	0.107 (B)
	0.047 (B)	0.039 (A)	0.034 (A)	0.035 (A)	0.015 (A)
St. Dev (%)	0.885 (F)	0.862 (F)	0.931 (F)	1.044 (F)	1.309 (F)
	0.856 (B)	0.867 (B)	0.941 (B)	1.053 (B)	1.457 (B)
	0.752(A)	0.729 (A)	0.926 (A)	0.940 (A)	1.122 (A)
Skewness	-2.859 (F)	-1.215 (F)	-1.082 (F)	-0.806 (F)	-0.354 (F)
	-0.065 (B)	-0.141 (B)	-0.186 (B)	-0.651 (B)	-0.410 (B)
	-0.680 (A)	-0.037 (A)	-1.524 (A)	-0.276 (A)	-0.218 (A)
Kurtosis	69.334 (F)	21.547 (F)	19.571 (F)	11.958 (F)	8.822 (F)
	5.278 (B)	3.778 (B)	7.892 (B)	7.560 (B)	9.333 (B)
	9.201 (A)	7.261 (A)	26.239 (A)	7.703 (A)	5.949 (A)
Normality [cv 6]	17287.0 (F)	6343.7 (F)	5907.4 (F)	2873.8 (F)	2121 (F)
	284.7 (B)	47.7 (B)	866.1 (B)	553.1 (B)	1130.5 (B)
	942.8 (A)	781.7 (A)	3700.4 (A)	843.4 (A)	430.8 (A)
ADF [cv - 2.87]	-25.88 (F)	-24.34 (F)	-24.34 (F)	- 24.06 (F)	-25.58 (F)
	-17.30 (B)	-17.86 (B)	-16.59 (B)	-17.06 (B)	-17.95 (B)
	-20.72 (A)	-18.21 (A)	-18.61 (A)	-17.26 (A)	-18.65 (A)

(F): 02/04/79 – 31/12/96; (B): 02/04/79 – 16/10/87; (A): 13/11/87 – 31/12/96. Source: Brunetti (1998)

Modelling the Mean (1): AR(p) & MA(q) Models

- AutoRegressive models - AR

- AR(1) $y_t = \mu + \phi_1 y_{t-1} + \varepsilon_t$

- AR(p) $y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t$

- Moving Average models - MA

- MA(1) $y_t = \mu + \theta_1 \varepsilon_{t-1} + \varepsilon_t$

- MA(q) $y_t = \mu + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots + \theta_q \varepsilon_{t-q} + \varepsilon_t$

Modelling the Mean (2): ARMA & ARIMA

- ARMA(p,q) Model - Box & Jenkins (1976)

$$\phi(L)(y_t - \mu) = \theta(L)\varepsilon_t$$
where μ is the mean of y_t , y_t is $I(0)$, ε_t is white noise and

$$\phi(L) = 1 - \sum_{j=1}^p \phi_j L^j, \quad \theta(L) = 1 + \sum_{j=1}^q \theta_j L^j$$

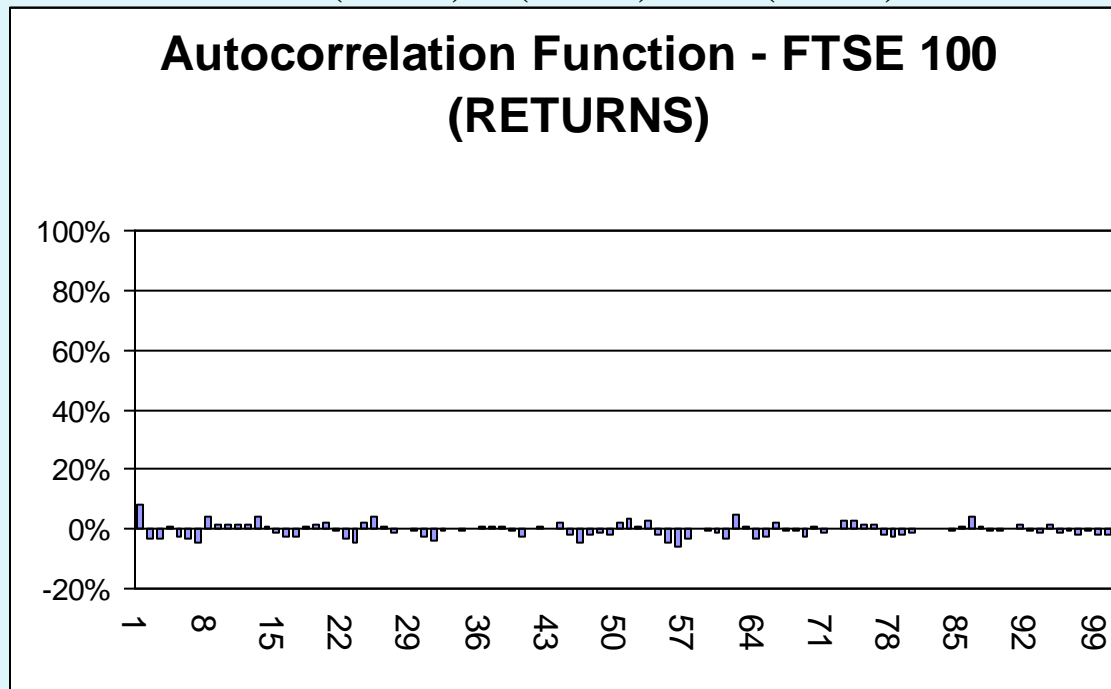
- If y_t is integrated of order $r \implies$ ARIMA(p,r,q) Model

$$\phi(L)(1-L)(y_t - \mu)^r = \theta(L)\varepsilon_t$$

Modelling the Mean (3): Empirical Application to FTSE 100 Returns

$$y_t = 0.0452 + 0.0023y_{t-1} + 0.0851\varepsilon_{t-1} + \varepsilon_t$$

(.0192) (.1578) (.5303)



Financial Market Volatility

- Volatility is the term used in finance for the day-to-day, week-to-week, month-to-month or year-to-year variation in asset price. It measures how much a price changes either about its constant long term level, or about a long term trend.
- A price can be very volatile but yet change very little in the long term; or alternatively show very little volatility but changes considerably over time.
- How can we estimate volatility?
- There are two main measures of volatility: Historical volatility and implied volatility
 - Historical volatility ==> backward looking.
 - Implied volatility ==> forward looking.

Historical Volatility (1)

- The most used measures of HV is *standard deviation* (Hull (1997)):
 - $n + 1$: number of observation
 - S_i : asset price at the end of period i
 - $u_i = \ln(S_i/S_{i-1})$ since $S_i = S_{i-1}\exp(u_i)$, u_i is the continuously compounded return.
- The standard deviation of u_i is given by

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (u_i - \bar{u})^2}$$

or

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n u_i^2 - \frac{1}{n(n-1)} \left(\sum_{i=1}^n u_i \right)^2}$$

- Problem: The choice of n is not easy.

Historical Volatility (2)

- More data generally lead to more accuracy ==> daily observation.
- Example: We have 20 daily data (a trading month) and we calculate the standard deviation $s = 0.0123$ ==> in order to have an estimate for the volatility per annum we need to multiply s by $(252)^{1/2}$:
 - ✦ $s = 0.0123 (252)^{1/2} = 0.195$ or 19.5%.
- Another important measure of HV is *squared returns*: $(u_i)^2$. Squared returns are variances (volatilities) of zero mean processes:
 - ✦ $s^2 = E(u^2) - [E(u)]^2$ ==> $E(u) = 0$.
- Squared returns are widely used because they provide a daily estimate of volatility. Moreover, the temporal structure (autocovariances) of squared returns is rich ==> ARCH-GARCH models.

Historical Volatility (3)

- Another important measure of HV is *absolute return*: $|u_i|$. Luce (1980) proved that absolute returns belong to a class of risk measure.
- Taylor (1987) first found that absolute returns of speculative assets have significant serial correlation over long lags. He also found that autocorrelations of absolute returns are greater than that of squared returns.
- Granger and Ding (1995) proved that absolute return and any power transformation of this return, may be interpret as a measure of risk.
- Empirical evidence shows that the power transformation of the absolute return $|u_i|^a$, has high autocorrelation for long lags and that this property is strongest when $a = 1$ [Ding, Granger and Engle (1993), Granger and Ding (1995), Ding and Granger (1996)].

Historical Volatility (5)

Parkinson (1980) and Garman & Klass (1980)

$$\hat{\sigma}_0^2 = (C_1 - C_0)^2, \quad \hat{\sigma}_1^2 = \frac{(O_1 - C_0)^2}{2f} + \frac{(C_1 - O_1)^2}{2(1-f)}$$

$$\hat{\sigma}_2^2 = \frac{(H_1 - L_1)^2}{4 \ln(2)} \quad (\text{Intraday Volatility})$$

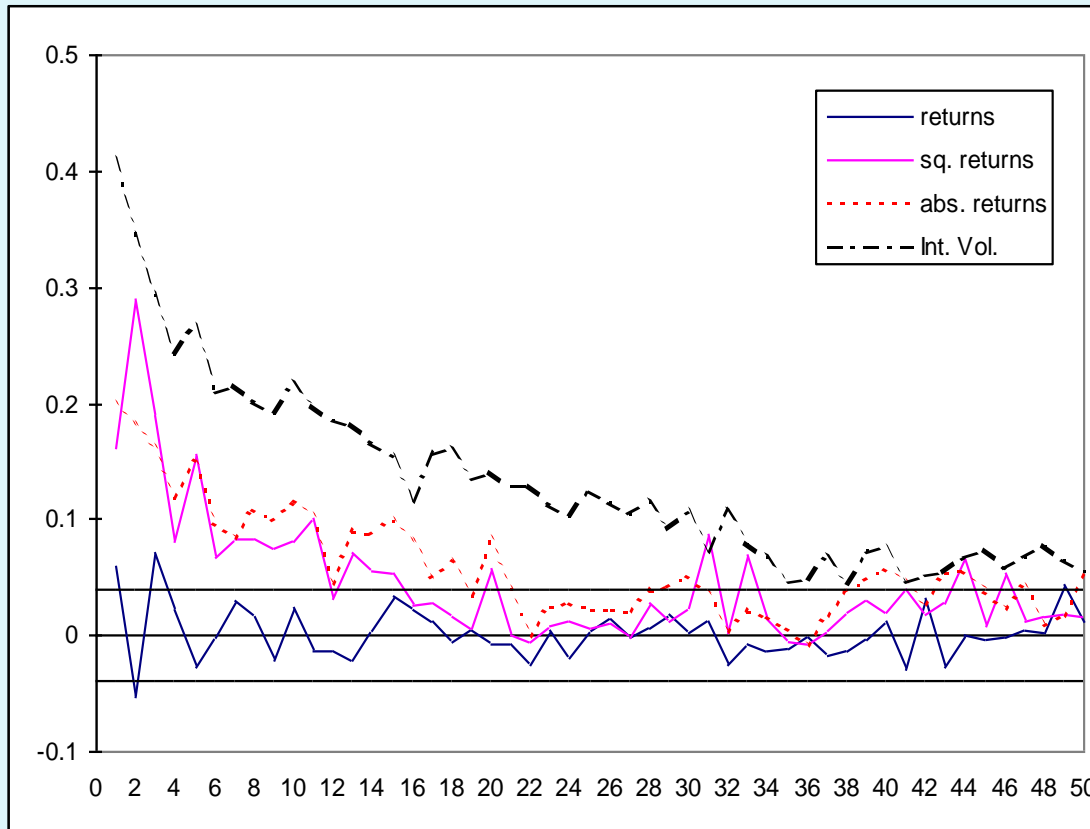
$$\hat{\sigma}_3^2 = a \frac{(O_1 - C_0)^2}{f} + (1-a) \frac{(u-d)^2}{(1-f)4 \ln(2)}$$

- Garman & Klass (180) show that extreme values estimators of volatility are much more efficient (lower variance) than close-close estimator (squared returns). However, discrete-time trading gives rise to a downward bias to intraday volatility. The bias is inversely related to trading volume. Beckers (1983) showed that high and low prices contain information unavailable in closing prices and this information is useful in forecasting volatility. Wiggins (1991) proved that intraday volatility is a much more efficient estimator than squared returns; however, he underlines the fact that recording errors may reverse this results.

Historical Volatility (6)

Parkinson (1980) and Garman & Klass (1980)

- Let us compare the autocorrelation functions of three measure of risk (volatility): Squared returns, absolute returns and intraday volatility ==> intraday volatility is estimated as follows:
$$\sigma_i = \frac{\ln(H_i) - \ln(L_i)}{2 \ln(2)}$$



Implied Volatility (1)

- Black-Scholes pricing formula:

$$c = SN(d_1) - Xe^{-r(T-t)}N(d_2)$$

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln(S/X) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}$$

- The only parameter in the B-S pricing formula that cannot be observed directly in the market is the volatility of the underlying asset (stock in our example) ==> we solve B-S to find volatility ==> *Implied volatility*.
- IV: It is the volatility implied by an option price observed in the market. Many analysts believe that IV represents the expectation of market participants on future volatility.

Implied Volatility (2)

- Is implied volatility a good estimate of volatility?
- Not really - Canina and Figlewski (1995)
- Yes - (many authors).

The Causes of Volatility

- It is possible to distinguish three main causes of financial market volatility:
 - Informational considerations - the arrival of new information in the market results in price adjustments as market agents evaluate the implications of the new information [French (1980)].
 - Hedging or speculative pressure - an increase in activity by one class of agents, say speculators, forces the price to adjust in such a way that the other group, in this case hedgers, can accommodate the increased speculative position. [Volatility is caused by trading.] [French and Roll (1986)]
 - (Only for commodities) The physical availability of the commodity - when a commodity is in short supply, a change in demand will have a larger price impact than when supplies are plentiful [Gilbert (1989)].

Modelling the Variance: ARCH-GARCH Models (1)

- AutoRegressive Conditional Heteroskedasticity: ARCH(q) - Engle (1982)

$$y_t = \mu_t + \varepsilon_t$$

$$\varepsilon_t = h_t^{1/2} e_t$$

$$e_t \sim N(0,1)$$

$$h_t \mid \Omega_{t-1} = \omega + \alpha(L)\varepsilon_t^2$$

where $\alpha(L)$ is a lag polynomial of order q , Ω_{t-1} is the information set available at time $t-1$ and y_t could be any regression function for the mean process

- The ARCH model enables us to distinguish between the conditional and the unconditional second order moment (variance). While the unconditional variance may be time invariant (homoskedasticity) or may not even exist, the conditional variance varies over time and depends on the past states of the world.

ARCH-GARCH Models (2)

- Bollerslev (1986) (see also Taylor, 1986) proposed the Generalised ARCH or GARCH(p,q) model

$$y_t = \mu_t + \varepsilon_t$$

$$\varepsilon_t = h_t^{1/2} e_t$$

$$e_t \sim N(0,1)$$

$$h_t | \Omega_{t-1} = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)h_t$$

- The GARCH model allows to estimate both the conditional mean and the conditional variance of a process.
- In the ARCH-GARCH class of models, the variance of the current error term is an increasing function of the magnitude of the past error term ==> large (small) errors are followed by large (small) errors. Hence these models are able to capture the volatility clustering phenomenon observed in financial data.
- Engle (1982) shows that the kurtosis of an ARCH(1) process is greater than 3 (the kurtosis of the normal distribution) ==> ARCH-GARCH models are able to capture the leptokurtosis behaviour of financial time series

ARCH-GARCH Models (3)

$$y_t = \mu + \theta \varepsilon_{t-1} + \varepsilon_t$$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

Table 2
MA(1)-GARCH(1,1) Estimation Results

	US	UK	Germany	France	Italy
μ (%)	0.054 (0.010)	0.062 (0.011)	0.044 (0.011)	0.07 (0.014)	0.056 (0.017)
θ	0.08 (0.014)	0.094 (0.015)	0.092 (0.015)	0.16 (0.015)	0.195 (0.014)
ω	0.007 (0.002)	0.018 (0.004)	0.018 (0.003)	0.041 (0.007)	0.035 (0.007)
α	0.082 (0.006)	0.081 (0.009)	0.087 (0.01)	0.103 (0.012)	0.092 (0.01)
β	0.848 (0.007)	0.891 (0.012)	0.89 (0.011)	0.855 (0.016)	0.89 (0.011)
$\alpha + \beta$	0.93	0.972	0.977	0.958	0.982

Standard errors in parentheses.
Source: Brunetti (1999)

ARCH-GARCH Models (4)

- Defining the innovations in the conditional variance process as

$$v_t = \varepsilon_t^2 - h_t$$

- The GARCH(p,q) model might be expressed as an **ARMA** process in v_t , $\phi(L)\varepsilon_t^2 = \omega + [1 - \beta(L)]v_t$

- Engle and Bollerslev (1986) considered the case of a unit root in the lag polynomial $\phi(L)$ and introduced the Integrated GARCH or IGARCH model

$$\phi(L)(1 - L)\varepsilon_t^2 = \omega + [1 - \beta(L)]v_t$$

- This equation may be interpreted as the **ARIMA** representation of the IGARCH process

Stochastic Volatility (1)

- Another response to non-normality of returns is to assume that there is a random variable (stochastic volatility) *conditional upon which* returns are normal. This variable (stochastic volatility) is not directly observed
- This kind of assumption is often made in continuous-time theoretical models where volatility is driven by a process separate from returns: there are two different processes, one for asset returns and one for volatility
- Example of stochastic volatility model

$$y_t = \varepsilon_t e^{\sigma_t/2}$$
$$\sigma_t = \varphi\sigma_{t-1} + \xi_t$$

$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$
$$\xi_t \sim N(0, \sigma_\xi^2)$$

Stochastic Volatility (2)



- Usually it is assumed that the two processes, ε_t and ξ_t , are serially uncorrelated and independent of each other
- If we square the return equation, y_t , and take logs, we get

$$\log(y_t^2) = \sigma_t + \log(\varepsilon_t^2)$$

$$\sigma_t = \varphi\sigma_{t-1} + \xi_t$$

- σ_t measures the difference between the conditional log standard deviation of returns and its mean; σ_t follows a zero-mean AR(1) process

Realized Volatility



- Integrated Volatility

$$\sigma_{t,t+\delta t}^2 = \int_t^{t+\delta t} \sigma^2(s) ds$$

$$\sigma_{RV,t}^2 = \sum_{i=1}^N r_i^2$$

$$r_i = \ln(P_t / P_{t-1})$$

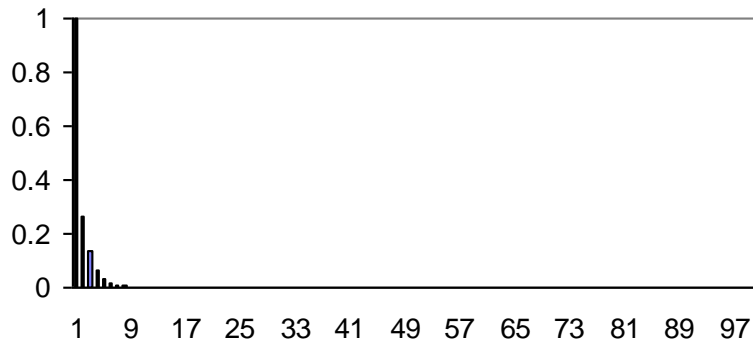
Long Memory (1)

- The phenomenon of long memory has been known since the time that ancient Egyptian hydrologists studied the flows and inflows of the river Nile. The idea is very simple and states that the effects of an event (shock) persist over a long period of time
- From an empirical point of view, the presence of long memory may be defined in terms of the observed autocorrelations, which show high dependence between very distant observations
- The autocorrelation function of a long memory process decays at an hyperbolic rate, while the autocorrelation function of an ARMA process decays exponentially

Long Memory (2)



Simulation of Autocorrelation Function for ARMA Models



Simulation of Autocorrelation Function for Long Memory Models

