CREDIT RISK

CREDIT RATINGS

- Rating Agencies: Moody's and S&P → Creditworthiness of corporate bonds
- In the S&P rating system, AAA is the best rating. After that comes AA, A, BBB, BB, B, and CCC
- The corresponding Moody's ratings are Aaa, Aa, A, Baa, Ba, B, and Caa
- Bonds with ratings of BBB (or Baa) and above are considered to be "investment grade"

HISTORICAL DATA

Historical data provided by rating agencies are also used to estimate the probability of default

CUMULATIVE AVE DEFAULT RATES (%) (1970-2003, MOODY'S)

	1	2	3	4	5	7	10
Aaa	0.00	0.00	0.00	0.04	0.12	0.29	0.62
Aa	0.02	0.03	0.06	0.15	0.24	0.43	0.68
A	0.02	0.09	0.23	0.38	0.54	0.91	1.59
Baa	0.20	0.57	1.03	1.62	2.16	3.24	5.10
Ba	1.26	3.48	6.00	8.59	11.17	15.44	21.01
B	6.21	13.76	20.65	26.66	31.99	40.79	50.02
Caa	23.65	37.20	48.02	55.56	60.83	69.36	77.91

INTERPRETATION

- The table shows the probability of default for companies starting with a particular credit rating-
- A company with an initial credit rating of Baa has a probability of 0.20% of defaulting by the end of the first year, 0.57% by the end of the second year, and so on-
- The probability that a bond rated Baa will default during the second year of its life is 0.57 – 0.30 = 0.37%
- For a company that starts with a good credit rating default probabilities tend to increase with time
- For a company that starts with a poor credit rating default probabilities tend to decrease with time

DEFAULT INTENSITIES VS UNCONDITIONAL DEFAULT PROBABILITIES

- The probability of default for a Caa bond in the 3rd year is:
  48.02 – 37.20 = 10.82% ➔ unconditional default probability ➔ it is the probability of default for a certain time period as seen at time zero
- The probability that the Caa-rated bond will survive until the end of year 2 is: 100 – 37.20 = 62.80%
- The probability that it will default during the 3rd year conditional on no earlier default is: 0.1082 / 0.6280 = 0.1723 ➔
- The default intensity (also called hazard rate) is the probability of default for a certain time period conditional on no earlier default

DEFAULT INTENSITIES

- We compute the default intensity (λ) for a year. Let's compute it for an interval of time Δt

  V(t): cumulative probability of the company surviving to time *t*

$$V(t + \Delta t) - V(t) = - \lambda(t)V(t)\Delta t$$

taking limits: $dV(t) / dt = - \lambda(t)V(t)$ ➔ $V(t) = \exp\left[-\int_0^t \lambda(\tau) d\tau \right]$

- Define Q(T) the probability of default at time *t*

$$Q(t) = 1 - \exp\left[-\int_0^t \lambda(\tau) d\tau \right] = 1 - \exp\left[\bar{\lambda}(t) t \right]$$

$\bar{\lambda}(t)$ is the average default intensity between time 0 and time t

RECOVERY RATE

When a company goes bankrupt, those that are owed money by the company file claims against the assets of the company

The recovery rate for a bond is usually defined as the price of the bond immediately after default as a percent of its face value

RECOVERY RATE

Recovery rates are significantly negatively correlated with default rates

Moody's looked at the average recovery rate and average defaults rates from 1982 till 2003 and found the following relationship

Average Recovery Rate = 50.3 – 6.3 $\times$ Average Default Rate

RECOVERY RATES
(MOODY'S: 1982 TO 2003)

Class	Mean(%)
Senior Secured	51.6
Senior Unsecured	36.1
Senior Subordinated	32.5
Subordinated	31.1
Junior Subordinated	24.5

ESTIMATING DEFAULT PROBABILITIES

Alternatives:

- Use Bond Prices
- Use spreads
- Use Historical Data
- Use Merton's Model

USING BOND PRICES

- The probability of default for a company can be estimated from the price of the bonds issued by the company ➔ the difference between the bond price and a similar risk-free bond captures the probability of default of the company

- This argument is ignoring liquidity: the lower the liquidity of a bond the lower the price ➔ however, it is a good approximation

USING BOND PRICES

Average default intensity over life of bond is approximately

$$h = \frac{s}{1 - R}$$

where *s* is the spread of the bond's yield over the risk-free rate and *R* is the recovery rate

- Example: assume that a bond yields 200 basis points more than a similar risk-free bond and that the expected recovery rate (R) in the event of default is 40%

  Probability of default = 0.02/(1 – 0.4) = 0.0333 or 3.33%

MORE EXACT CALCULATION

- Assume that a five year corporate bond pays a coupon of 6% per annum (semiannually). The yield is 7% with continuous compounding and the yield on a similar risk-free bond is 5% (with continuous compounding)
- Price of risk-free bond is 104.09; price of corporate bond is 95.34; expected loss from defaults is 8.75
- Suppose that the probability of default is *Q* per year and that defaults always happen half way through a year (immediately before a coupon payment)

CALCULATIONS

Time (yrs)	Def Prob	Recovery Amount	Risk-free Value	Loss Given Default	Discount Factor	PV of Exp Loss
0.5	Q	40	106.73	66.73	0.9753	65.08Q
1.5	Q	40	105.97	65.97	0.9277	61.20Q
2.5	Q	40	105.17	65.17	0.8825	57.52Q
3.5	Q	40	104.34	64.34	0.8395	54.01Q
4.5	Q	40	103.46	63.46	0.7985	50.67Q
Total						288.48Q

CALCULATIONS

- We set 288.48Q = 8.75 to get Q = 3.03%
- This analysis can be extended to allow defaults to take place more frequently
- With several bonds we can use more parameters to describe the default probability distribution

THE RISK-FREE RATE

- The risk-free rate when default probabilities are estimated is usually assumed to be the LIBOR/swap zero rate (or sometimes 10 bps below the LIBOR/swap rate)
- To get direct estimates of the spread of bond yields over swap rates we can look at asset swaps

REAL WORLD VS RISK-NEUTRAL DEFAULT PROBABILITIES

- The default probabilities backed out of bond prices or credit default swap spreads are risk-neutral default probabilities
- The default probabilities backed out of historical data are real-world default probabilities

A COMPARISON

- Calculate 7-year default intensities from the Moody's data (These are real world default probabilities)
- Use Merrill Lynch data to estimate average 7-year default intensities from bond prices (these are risk-neutral default intensities)
- Assume a risk-free rate equal to the 7-year swap rate minus 10 basis point

DEFAULT INTENSITIES FROM THE MOODY'S DATA

- These are derived from Table 1

$$\bar{\lambda}(7) = -\frac{1}{7}\ln[1 - Q(7)]$$

$$\text{for A - ratyed company } Q(7) = 0.0091$$

$$\bar{\lambda}(7) = -\frac{1}{7}\ln[0.9909] = 0.0013$$

DEFAULT INTENSITIES FROM BOND PRICE

- Based on bond yields published by Merrill Lynch

  Recovery rate: R = 40%

  A-rated bonds, average Merrill Lynch yield: 6.274%

  Average swap rate: 5.605% less 10 basis points → 5.505%

  Average 7-year default probability

$$(0.06274 – 0.05505) / (1 – 0.4) = 0.0128$$

REAL WORLD VS RISK NEUTRAL DEFAULT PROBABILITIES, 7 YEAR AVERAGES

Rating	Real-world default probability per yr (bps)	Risk-neutral default probability per yr (bps)	Ratio	Difference
Aaa	4	67	16.8	63
Aa	6	78	13.0	72
A	13	128	9.8	115
Baa	47	238	5.1	191
Ba	240	507	2.1	267
B	749	902	1.2	153
Caa	1690	2130	1.3	440

RISK PREMIUMS EARNED BY BOND TRADERS

Rating	Bond Yield Spread over Treasuries (bps)	Spread of risk-free rate used by market over Treasuries (bps)	Spread to compensate for default rate in the real world (bps)	Extra Risk Premium (bps)
Aaa	83	43	2	38
Aa	90	43	4	43
A	120	43	8	69
Baa	186	43	28	115
Ba	347	43	144	160
B	585	43	449	93
Caa	1321	43	1014	264

POSSIBLE REASONS FOR THESE RESULTS

- Corporate bonds are relatively illiquid
- The subjective default probabilities of bond traders may be much higher than the estimates from Moody's historical data
- Bonds do not default independently of each other. This leads to systematic risk that cannot be diversified away.
- Bond returns are highly skewed with limited upside. The non-systematic risk is difficult to diversify away and may be priced by the market

WHICH WORLD SHOULD WE USE?

- We should use risk-neutral estimates for valuing credit derivatives and estimating the present value of the cost of default
- We should use real world estimates for calculating credit VaR and scenario analysis

MERTON'S MODEL

- Merton's model regards the equity as an option on the assets of the firm

  V_0: Value of the company assets today; V_T: Value of the company assets at T;

  E_0: Value of the company equity today; E_T: Value of the company equity at T;

  D: debt, interest plus principal due to be paid at time T;

  σ_V: volatility of asset (assumed to be constant)

  σ_E: volatility of equity

  Two scenarios

1. $V_T < D$ → the company will default → $E_T = 0$
2. $V_T > D$ → the company will be able to repay the debt → $E_T = V_T - D$

The equity value is $\max(V_T - D, 0)$

EQUITY VS. ASSETS

An option pricing model enables the value of the firm's equity today, E_0, to be related to the value of its assets today, V_0, and the volatility of its assets, σ_V

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \; ; \quad d_2 = d_1 - \sigma_V \sqrt{T}$$

VOLATILITIES

$$\sigma_E E_0 = \frac{\partial E}{\partial V} \sigma_V V_0 = N(d_1) \sigma_V V_0$$

This equation together with the option pricing relationship enables V_0 and σ_V to be determined from E_0 and σ_E

EXAMPLE

- A company's equity is \$3 million and the volatility of the equity is 80%
- The risk-free rate is 5%, the debt is \$10 million and time to debt maturity is 1 year
- Solving the two equations yields V_0=12.40 and σ_v=21.23%

EXAMPLE CONTINUED

- The probability of default is $N(-d_2)$ or 12.7%

- The market value of the debt is: V_0 - E_0 = 12.4 – 3 = 9.40

- The present value of the promised payment is:
  10exp[-0.05*1] = 9.51

- The expected loss on the debt is: (9.51 – 9.40)/9.51 = 1.2%

- The recovery rate is: (12.7 – 1.2)/12.7 = 91%

THE IMPLEMENTATION OF MERTON'S MODEL (E.G. MOODY'S KMV)

- Choose time horizon
- Calculate cumulative obligations to time horizon. This is termed by KMV the "default point". We denote it by *D*
- Use Merton's model to calculate a theoretical probability of default
- Use historical data or bond data to develop a one-to-one mapping of theoretical probability into either real-world or risk-neutral probability of default.

CREDIT RISK IN DERIVATIVES TRANSACTIONS

The credit exposure on a derivative transaction is more complicated than that of a loan

Three cases

1. Contract always an asset (example: short option position)
2. Contract always a liability (example: long option position)
3. Contract can be an asset or a liability (example: forward contract)

For (1) there is no credit risk → if the counterparty goes bankrupt, there will be no loss → the derivative is an asset for the counterparty

For (2) there is always credit risk → if the counterparty goes bankrupt, there will be a loss → the derivative is a liability for the counterparty

For (3), it depends, if asset → no credit risk; if liability → credit risk

GENERAL RESULT

- Assume that default probability is independent of the value of the derivative
- Consider times $t_1, t_2, \ldots t_n$ and default probability is q_i at time t_i. The value of the contract at time t_i is f_i and the recovery rate is R
- The loss from defaults at time t_i is $q_i(1\text{-}R)E[\max(f_i,0)]$.
- Defining $u_i=q_i(1\text{-}R)$ and v_i as the value of a derivative that provides a payoff of $\max(f_i,0)$ at time t_i, the cost of defaults is

$$\sum_{i=1}^{n} u_i v_i$$

CREDIT RISK MITIGATION

- Netting
- Collateralization
- Downgrade triggers

DEFAULT CORRELATION

- The credit default correlation between two companies is a measure of their tendency to default at about the same time
- Default correlation is important in risk management when analyzing the benefits of credit risk diversification
- It is also important in the valuation of some credit derivatives, eg a first-to-default CDS and CDO tranches.

MEASUREMENT

- There is no generally accepted measure of default correlation
- Default correlation is a more complex phenomenon than the correlation between two random variables

BINOMIAL CORRELATION MEASURE

- One common default correlation measure, between companies *i* and *j* is the correlation between
  - A variable that equals 1 if company *i* defaults between time 0 and time *T* and zero otherwise
  - A variable that equals 1 if company *j* defaults between time 0 and time *T* and zero otherwise
- The value of this measure depends on *T*. Usually it increases at *T* increases.

BINOMIAL CORRELATION

Denote $Q_i(T)$ as the probability that company *A* will default between time zero and time *T*, and $P_{ij}(T)$ as the probability that both *i* and *j* will default. The default correlation measure is

$$\beta_{ij}(T) = \frac{P_{ij}(T) - Q_i(T)Q_j(T)}{\sqrt{[Q_i(T) - Q_i(T)^2][Q_j(T) - Q_j(T)^2]}}$$

SURVIVAL TIME CORRELATION

- Define t_i as the time to default for company *i* and $Q_i(t_i)$ as the probability distribution for t_i
- The default correlation between companies *i* and *j* can be defined as the correlation between t_i and t_j
- But this does not uniquely define the joint probability distribution of default times

GAUSSIAN COPULA MODEL

- Define a one-to-one correspondence between the time to default, t_i, of company i and a variable x_i by

$$Q_i(t_i) = N(x_i) \quad \text{or} \quad x_i = N^{-1}[Q(t_i)]$$

where N is the cumulative normal distribution function.

- This is a "percentile to percentile" transformation. The p percentile point of the Q_i distribution is transformed to the p percentile point of the x_i distribution. x_i has a standard normal distribution
- We assume that the x_i are multivariate normal. The default correlation measure, ρ_{ij} between companies i and j is the correlation between x_i and x_j

BINOMIAL VS GAUSSIAN COPULA MEASURES

The measures can be calculated from each other

$$P_{ij}(T) = M[x_i, x_j; \rho_{ij}]$$

so that

$$\beta_{ij}(T) = \frac{M[x_i, x_j; \rho_{ij}] - Q_i(T)Q_j(T)}{\sqrt{[Q_i(T) - Q_i(T)^2][Q_j(T) - Q_j(T)^2]}}$$

where M is the cumulative bivariate normal probability distribution function

COMPARISON

- The correlation number depends on the correlation metric used
- Suppose $T = 1$, $Q_i(T) = Q_j(T) = 0.01$, a value of ρ_{ij} equal to 0.2 corresponds to a value of $\beta_{ij}(T)$ equal to 0.024.
- In general $\beta_{ij}(T) < \rho_{ij}$ and $\beta_{ij}(T)$ is an increasing function of T

EXAMPLE OF USE OF GAUSSIAN COPULA

Suppose that we wish to simulate the defaults for *n* companies . For each company the cumulative probabilities of default during the next 1, 2, 3, 4, and 5 years are 1%, 3%, 6%, 10%, and 15%, respectively

USE OF GAUSSIAN COPULA CONTINUED

- We sample from a multivariate normal distribution to get the x_i
- Critical values of x_i are

$N^{-1}(0.01) = -2.33$, $N^{-1}(0.03) = -1.88$,

$N^{-1}(0.06) = -1.55$, $N^{-1}(0.10) = -1.28$,

$N^{-1}(0.15) = -1.04$

USE OF GAUSSIAN COPULA

- When sample for a company is less than -2.33, the company defaults in the first year
- When sample is between -2.33 and -1.88, the company defaults in the second year
- When sample is between -1.88 and -1.55, the company defaults in the third year
- When sample is between -1,55 and -1.28, the company defaults in the fourth year
- When sample is between -1.28 and -1.04, the company defaults during the fifth year
- When sample is greater than -1.04, there is no default during the first five years

A ONE-FACTOR MODEL FOR THE CORRELATION STRUCTURE

$$x_i = a_i M + \sqrt{1 - a_i^2}\, Z_i$$

- The correlation between x_i and x_j is $a_i a_j$
- The ith company defaults by time T when $x_i < N^{-1}[Q_i(T)]$ or

$$Z_i < \frac{N^{-1}[Q_i(T) - a_i M]}{\sqrt{1 - a_i^2}}$$

- The probability of this is

$$Q_i(T|M) = N\left\{\frac{N^{-1}[Q_i(T)] - a_i M}{\sqrt{1 - a_i^2}}\right\}$$

CREDIT VAR

- Can be defined analogously to Market Risk VaR
- A *T*-year credit VaR with an *X*% confidence is the loss level that we are *X*% confident will not be exceeded over *T* years

CREDITMETRICS (PAGE 500-502)

- Calculates credit VaR by considering possible rating transitions
- A Gaussian copula model is used to define the correlation between the ratings transitions of different companies