OPTIONS

OUTLINE

- Definition
- Payoffs
- Mechanics
- Other option-type products

DEFINITIONS AND TERMINOLOGIES

- An option gives the option holder the right/option, but no obligation, to buy or sell a security to the option writer/seller
    - for a pre-specified price (the strike price, K)
    - at (or up to) a given time in the future (the expiry date or maturity date)
- An option has positive value.

  Comparison: a forward contract has zero value at inception.

OPTION TYPES

- A call option gives the holder the right to buy a security. The payoff is

  $(S_T - K)^+$ when exercised at maturity.

  - If the stock price is less than strike price, the investor clearly choose not to exercise, the investor looses only the option premium.

- A put option gives the holder the right to sell a security. The payoff is

  $(K - S_T)^+$ when exercised at maturity.

  - If the stock price is more than strike price, the investor clearly choose not to exercise and looses only the option premium.

- American options can be exercised at any time priory to expiry.
- European options can only be exercised at the expiry.
- Bermudan option can only be exercised during specified period

MORE TERMINOLOGIES

- Strike or Exercise Price: The price at which the futures contract underlying an option can be purchased (if a call) or sold (if a put). In the call and put definitions above, this is the predetermined price.
- Premium: The price paid by the buyer to the seller to purchase an option. This price is arrived at through trading on an exchange market, as with futures.
- Purchaser or holder: The person who buys a call or a put option and pays the option premium, i.e. the person who establishes a long options position. This is the party with the right, but not the obligation, under the terms of the contract.

MORE TERMINOLOGIES

- Grantor: The person who sells a call or put option and receives the option premium, i.e. the person who establishes a short position. This party is obligated to perform under the terms of option.
- Exercise: The exercise of a call gives the the option purchaser a long position in the underlying futures contract at the option's strike price; the exercise of a put option gives the option purchaser a short futures position at the option's strike price.

CALL OPTION

- Call option is a contract where the buyer has the right to buy, but not the obligation.
- Since buyer decide whether to buy, the seller cannot make money at expiration. To take this risk, the seller is compensated by option premium, which is agreed when the contract signed.
- Example: Consider a call option on the S&R index with 6 months to expiration and strike price of $1000 and premium of $93.81. And assume that the risk free rate is 2% over 6 months. Suppose that the index in 6 months is $1,100. Clearly it is worthwhile to pay the $1000 strike price to acquire the index worth $1100. If on the other hand the index is 900 at expiration, it is not worthwhile paying the $1000 strike price to buy the index worth $900.

CALL OPTION

- Remember the buyer is not obliged to buy the index and hence will only exercise the option if the payoff is positive.

$$\text{Purchased call payoff} = \max(0, S_T - K)$$

  - In our example, K=1000. If S=1100 then the call payoff

$$\text{Purchased call payoff} = \max(0, 1100-1000) = \$100$$

  - If S=900, then the call payoff is

$$\text{Purchased call payoff} = \max(0, 900-1000) = \$0$$

- Payoff does not take into account of the initial cost of aquiring the position. For a purchased option, the premium is paid at the time the option is acquired. In computing profit at expiration, we use future value of the premium.

$$\textbf{Purchased call profit} = \max(0, S_T - K) - \textbf{future value of option premium}$$

Purchased call profit=Purchased call payoff-future value of option premium

PROFIT FROM CALL OPTION

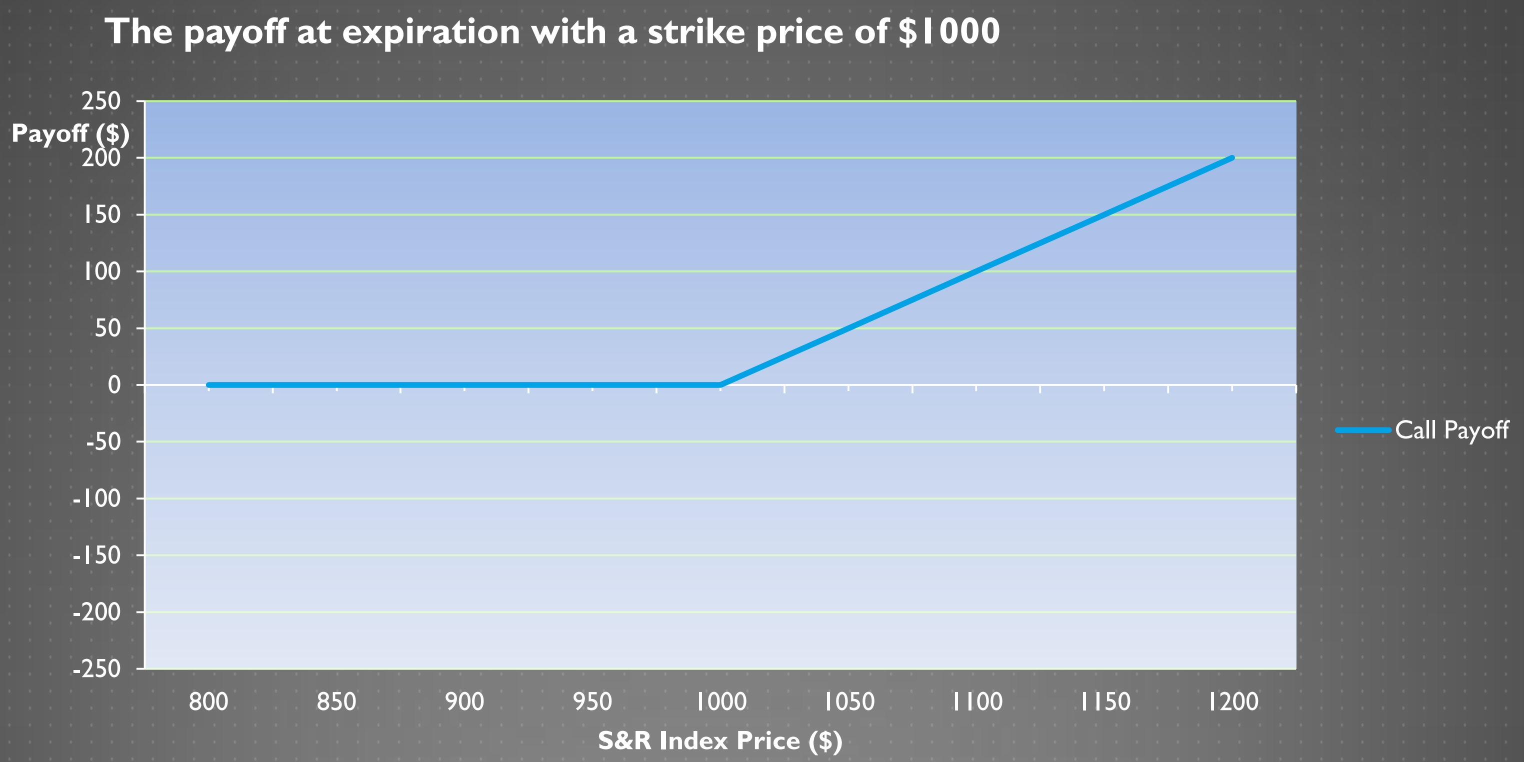
- If the index at the expiration is 1100, then profit is

  Purchased call profit=max(0,1100-1000)-93.81*1.02=$4.32

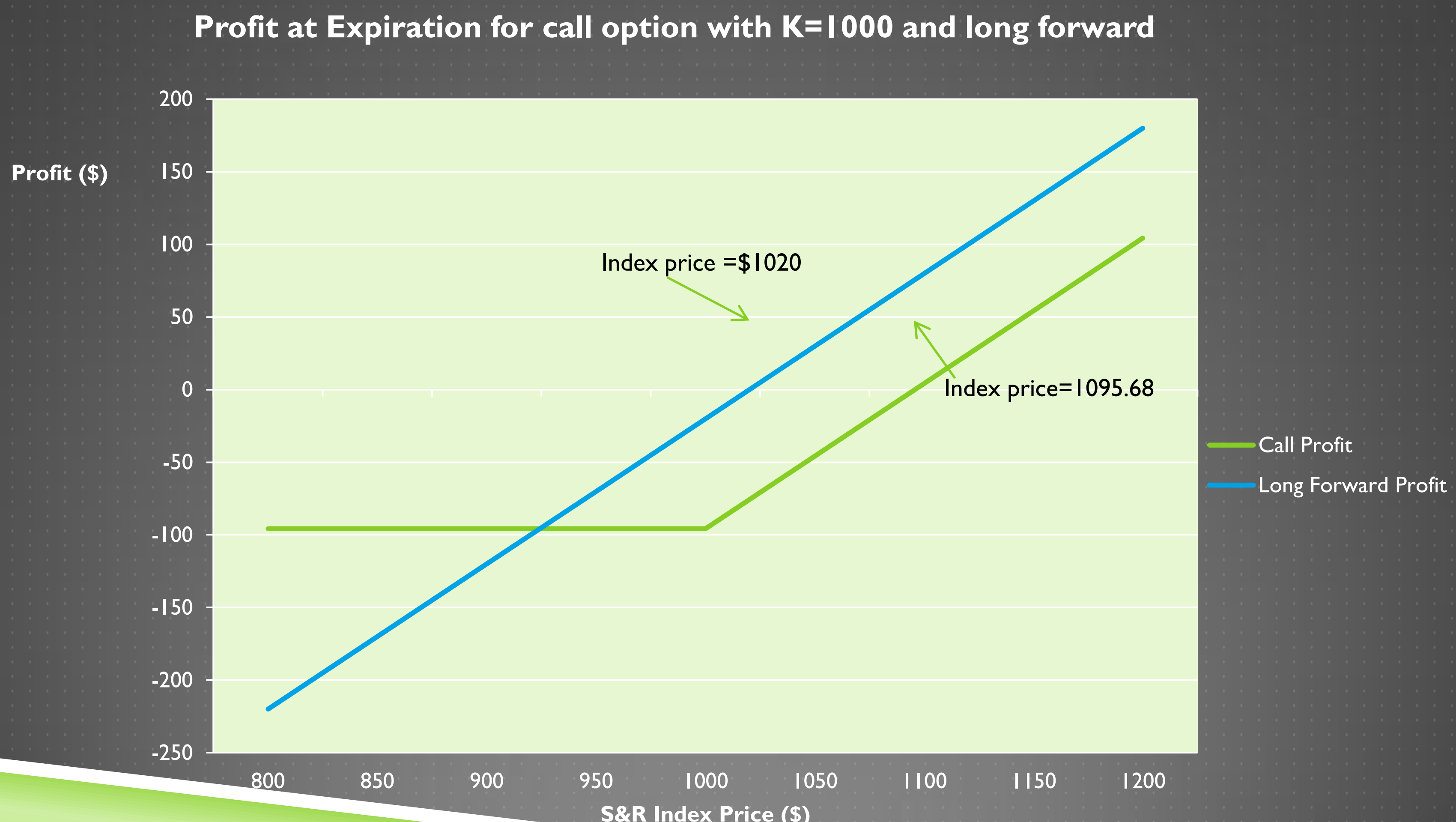
- If the index at the expiration is 900, then the owner does not exercise the option. The loss will be future value of option premium. Maximum loss will be the option premium.

  Purchased call profit=max(0,900-1000)-93.81*1.02=-$95.68

PAYOFF FROM CALL OPTION



PROFIT FROM CALL OPTION AND LONG FORWARD



PAYOFF AND PROFIT FOR A WRITTEN CALL OPTION

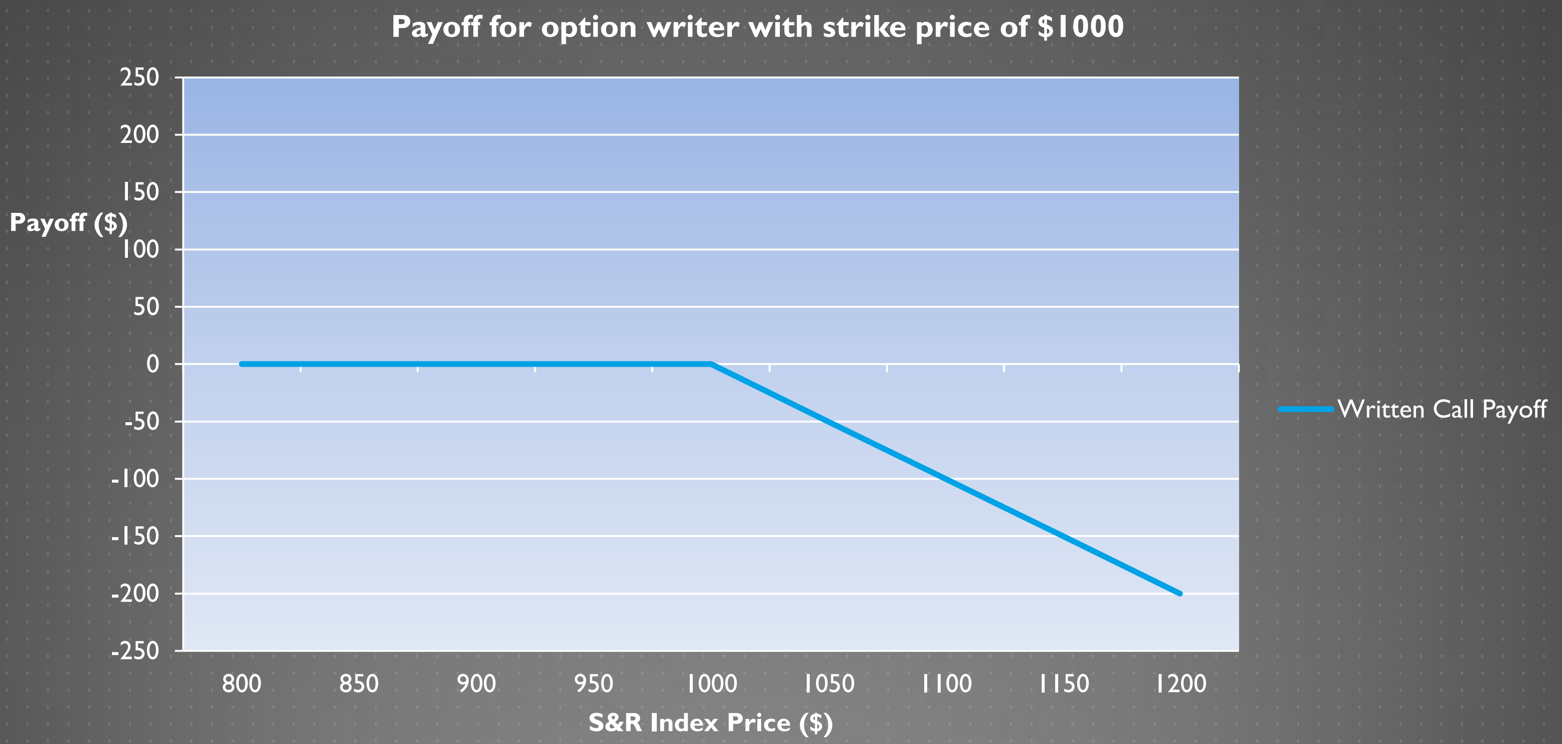
- Option writer (seller of option) has a short position in a call option. The writer receives the premium for the option and then has an obligation to sell the underlying security in exchange for the strike price if the option buyer exercises the option.
- The payoff and profit to a written call are just the opposite of those for a purchased call.

$$\text{Written call payoff} = -\max(0, S_T - K) = \min(0, K - S_T)$$

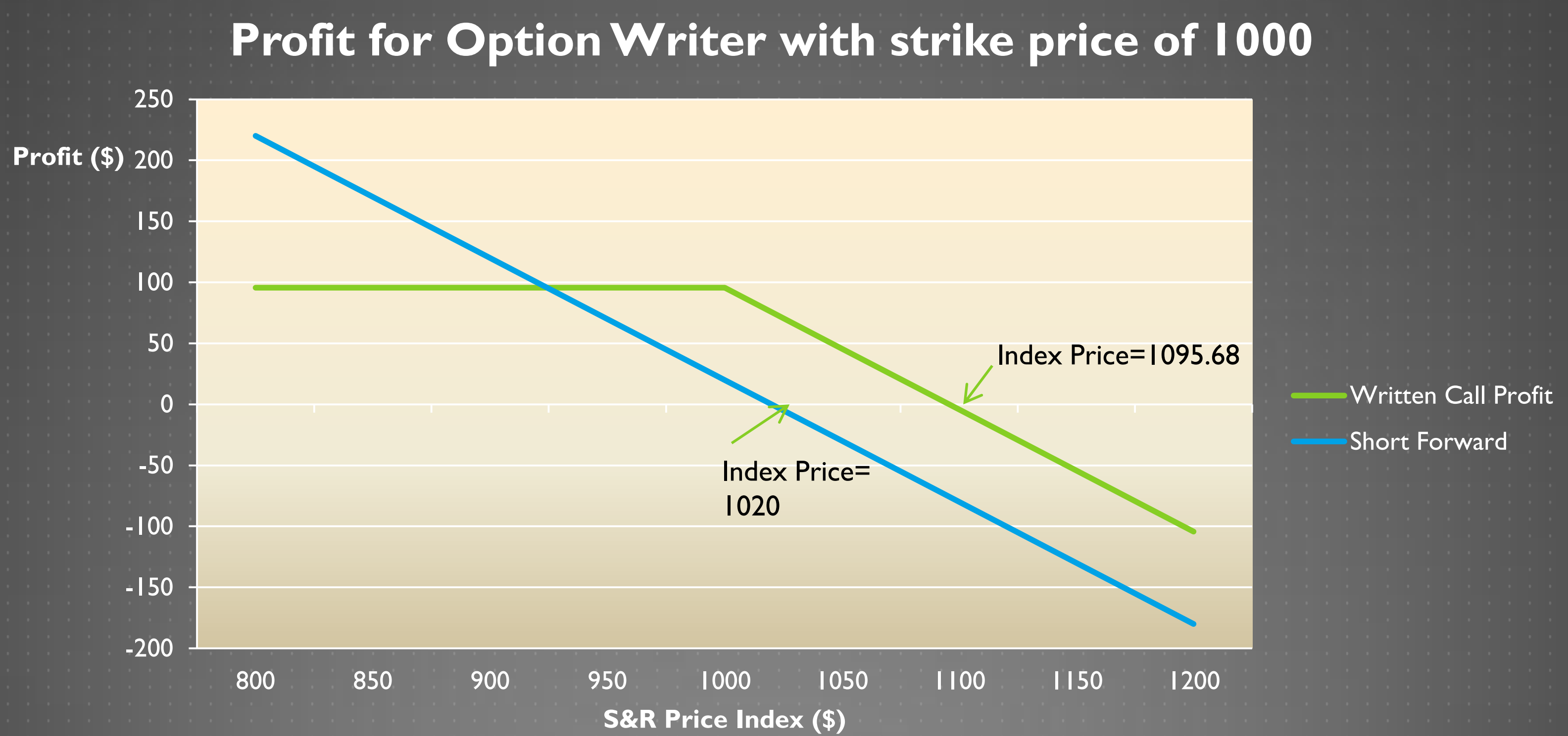
$$\text{Written call profit} = -\max(0, S_T - K) + \text{future value of option premium}$$

- In our example, if S=1100 then option writer payoff will be -100$ and profit will be -$4.32. If on the other hand, S=900, then payoff will be 0 and profit will be the future value of premium, $95.68

PAYOFF FOR OPTION WRITER



PROFIT FOR OPTION WRITER



PUT OPTION

- Put option is a contract where the buyer has the right to sell, but not the obligation.
- Since buyer decide whether to sell, the seller cannot make money at expiration. To take this risk, the seller is compensated by option premium, which is agreed when the contract signed.
- Example: Consider a put option on the S&R index with 6 months to expiration and strike price of $1000 and premium of $74.20. And assume that the risk free rate is 2% over 6 months. Suppose that the index in 6 months is $1,100. Clearly it is not worthwhile to sell the index worth $1100 for the strike price of $1000. If on the other hand the index is 900 at expiration, it is worthwhile selling the index for $1000.

PUT OPTION

- Remember the buyer is not obliged to sell the index and hence will only exercise the option if the payoff is positive.

$$\text{Purchased put payoff} = \max(0, K - S_T)$$

  - In our example, K=1000. If S=1100 then the put payoff

    Purchased put payoff=max(0,1000-1100)=$0

  - If S=900, then the put payoff is

    Purchased put payoff=max(0,1000-900)=$100

- Payoff does not take into account of the initial cost of acquiring the position. For a purchased option, the premium is paid at the time the option is acquired. In computing profit at expiration, we use future value of the premium.

$$\textbf{Purchased put profit} = \max(0, K - S_T) - \textbf{future value of option premium}$$

Purchased put profit=Purchased put payoff-future value of option premium

PROFIT FROM PUT OPTION

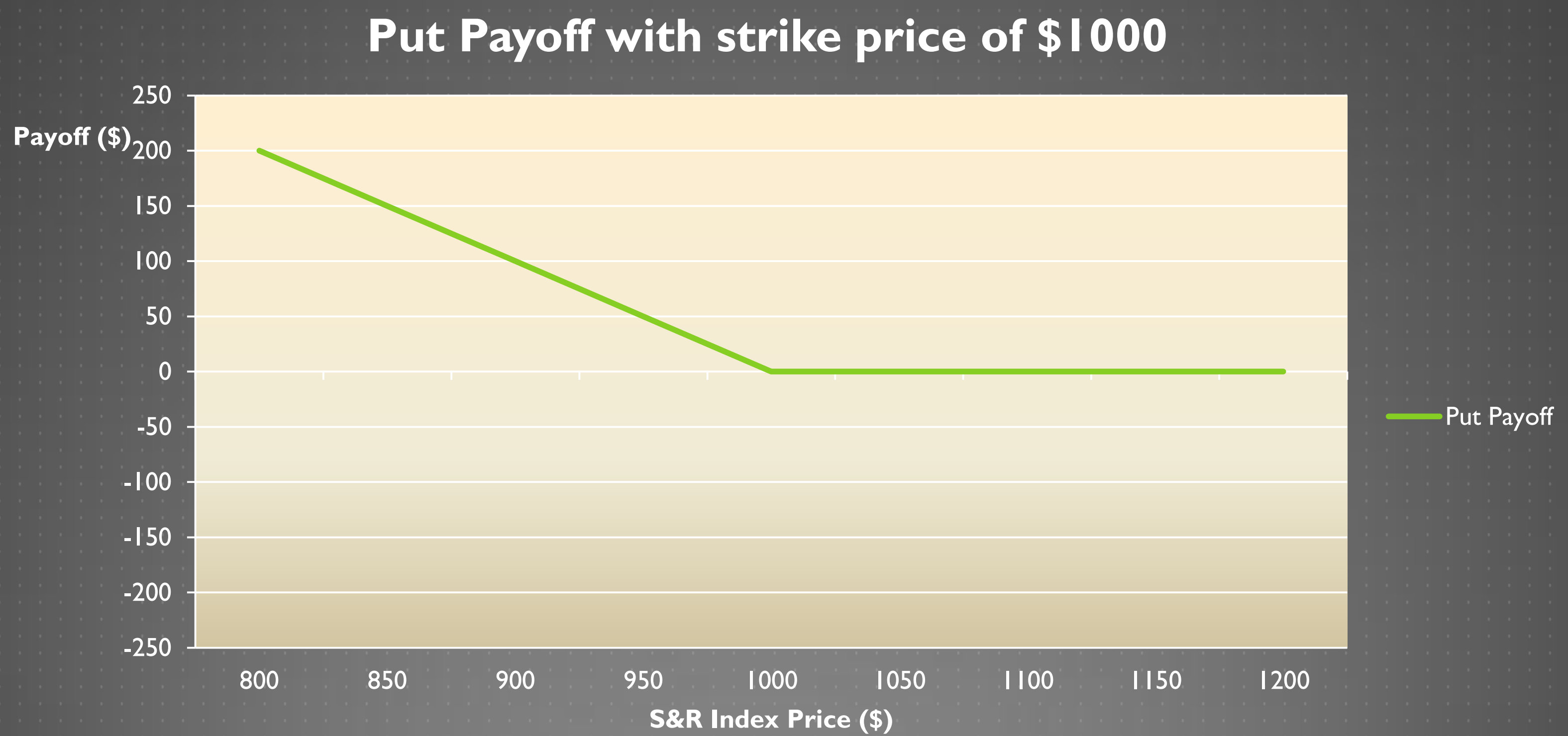
- If the index at the expiration is 1100, then option buyer will not exercise and maximum loss will be the future value of option premium.

  Purchased put profit=max(0,1000-1100)-74.2*1.02=-$75.68

- If the index at the expiration is 900, then the owner exercise the option. The profit will be

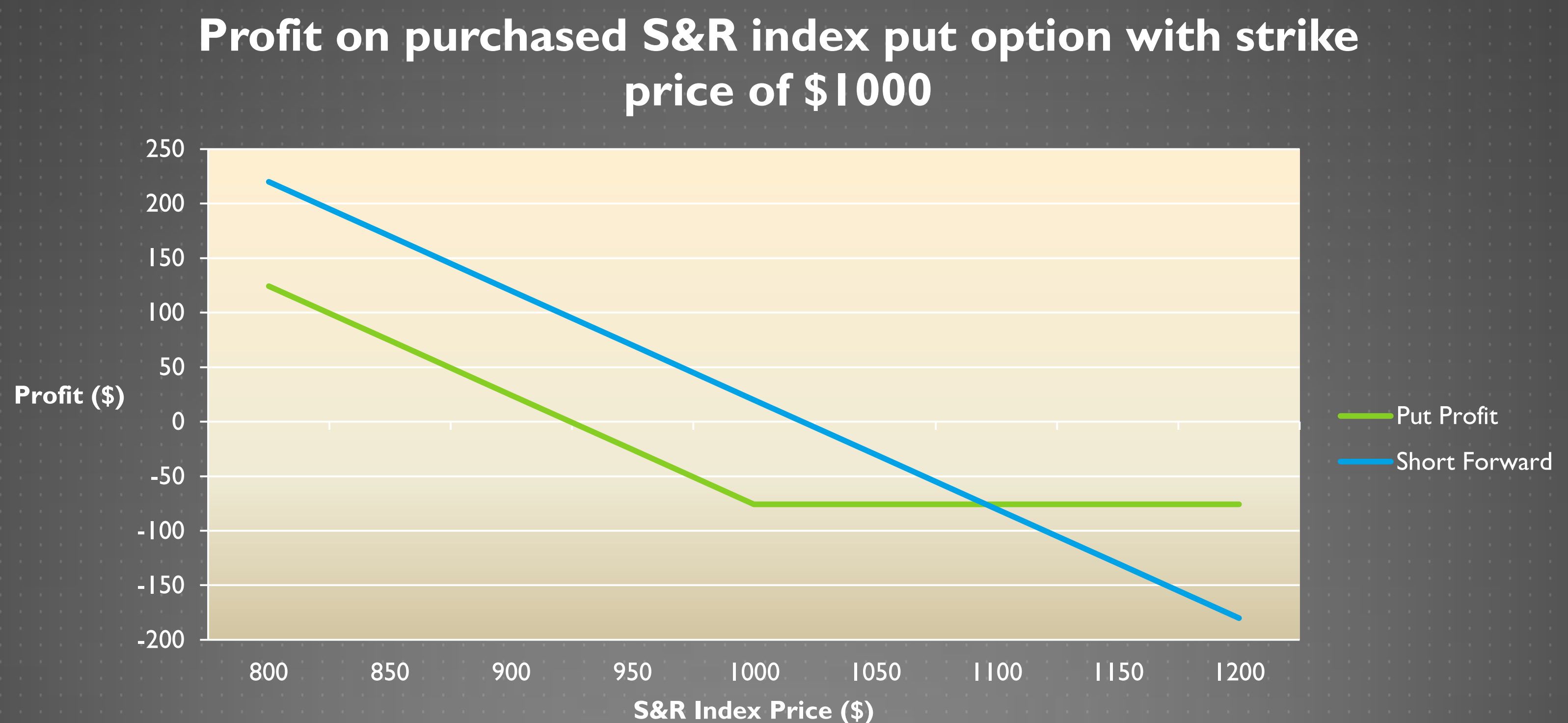
  Purchased put profit=max(0,1000-900)-74.2*1.02=$24.32

PUT PAYOFF



Put Payoff with strike price of $1000
Payoff ($)
250
200
150
100
50
0
-50
-100
-150
-200
-250
800
850
900
950
1000
1050
1100
1150
1200
S&R Index Price ($)
Put Payoff

PURCHASED PUT PROFIT



PAYOFF AND PROFIT FOR A WRITTEN PUT OPTION

- Option writer (seller of option) has a long position in a put option. The writer receives the premium for the option and then has an obligation to buy the underlying security in exchange for the strike price if the option buyer exercises the option.
- The payoff and profit to a written put are just the opposite of those for a purchased put.

  Written put payoff=-max(0,K-S_T)=min(0,S_T-K)

  Written put profit=-max(0,K-S_T)+future value of option premium

- In our example, if S=1100 then put buyer will not exercise the put, thus put writer earn profit, which will be option premium. If, on the other hand, S=900, then option buyer exercise option and option seller (writer) will lose $24.32 ((-100+$75.68)

MONEYNESS

- **Moneyness:** the strike relative to the spot/forward level
  - An option is said to be in-the-money if the option has positive value if exercised right now:
    - $S_t > K$ for call options and $S_t < K$ for put options. Sometimes it is also defined in terms of the forward price at the same maturity (in the money forward): $F_t > K$ for call and $F_t < K$ for put
    - The option has positive intrinsic value (defined as the maximum of zero and the value of the option would have if it is exercised today) when in the money. The intrinsic value is $(S_t - K)^+$ for call, $(K - S_t)^+$ for put. We can also define intrinsic value in terms of forward price.

MONEYNESS

- An option is said to be out-of-the-money when it has zero intrinsic value.
  - $S_t < K$ for call options and $S_t > K$ for put options. Out-of-the-money forward: $F_t < K$ for call and $F_t > K$ for put.
- An option is said to be at-the-money spot (or forward) when the strike is equal to the spot (or forward).

OPTION POSITIONS

Long Positions: Long forward, purchased call, written put (All these positions benefit from rising prices)

Short Positions: Short forward, purchased put, written call (All these positions benefit from falling prices)

Position	Maximum Loss	Maximum Gain
Long Forward	-Forward Price	Unlimited
Short Forward	Unlimited	Forward Price
Long Call (Purchased call)	-FV(premium)	Unlimited
Short Call (Written call)	Unlimited	FV(premium)
Long Put (Purchased put)	-FV(premium)	K-FV(premium)
Short Put (Written put)	FV(premium)-K	FV(premium)

OPTIONS ON STOCKS

- Margins: Options *writers* are always margined. Options buyers used to pay the option price upfront and were not margined unless maturity of option is greater than 9 months. For options with maturities greater than 9 months investors can buy on margin, borrowing up to 25% of the option value
- This system may still operate for some OTCs. Exchange option purchases are now margined as for futures.
- Options may be exchange options:
  - Stock options (Chicago Board Options Exchange (CBOE) and NYSE)
  - Foreign Currency options (Philadelphia Exchange (PHLK))
  - Index options (S&P 100, S&P 500 on CBOE)

OPTIONS ON STOCKS

- A naked option is an option that is not combined with an offsetting position in the underlying stock.
- Margin on naked call option is the greater of
  - A total of 100% of the proceeds of the sale plus 20% of the underlying share price minus the amount, if any, by which the option is out of money
  - A total of 100%of the option proceeds plus 10% of the underlying share price
- Margin on naked put option is the greater of
  - A total of 100% of the proceeds of the sale plus 20% of the underlying share price minus the amount, if any, by which the option is out of money
  - A total of 100%of the option proceeds plus 10% of the exercise price

OPTIONS ON FUTURES

- Futures options (the futures contract normally matures shortly after the expiration of the option. When the holder of a call option exercises, he/she acquires a long position in the underlying futures contract plus a cash amount equal to the excess of the futures price over the strike price ==> the futures contract has zero value - Chicago Board of Trade, Treasury bond futures options)
- Over-the-counter Options (OTC)
  - OTC markets: Financial institutions and corporations trade directly with each other (foreign currencies & interest rates).
  - Advantage: The OTC option can be designed to suit the needs of the parties involved - non standard features can be incorporated into the design of the option.
  - *Bermudan* option: It is exercisable only on certain specific days.
  - *Asian* option: the payoff is designed in terms of the average value of the underlying asset during a certain time period rather than in terms of its final value.

OTHER OPTIONS

- Warrants
- Employee Stock Options
- Convertibles

WARRANTS

- Warrants are options that are issued (or written) by a corporation or a financial institution
- The number of warrants outstanding is determined by the size of the original issue and changes only when they are exercised or when they expire
- Warrants are traded in the same way as stocks
- The issuer settles up with the holder when a warrant is exercised
- When call warrants are issued by a corporation on its own stock, exercise will lead to new treasury stock being issued

EXECUTIVE STOCK OPTIONS

- Option issued by a company to executives
- When the option is exercised the company issues more stock
- Usually at-the-money when issued
- They become vested after a period of time
- They cannot be sold
- They often last for as long as 10 or 15 years

CONVERTIBLE BONDS

- Convertible bonds are regular bonds that can be exchanged for equity at certain times in the future according to a predetermined exchange ratio
- Very often a convertible is callable
- The call provision is a way in which the issuer can force conversion at a time earlier than the holder might otherwise choose

ASSETS UNDERLYING EXCHANGE-TRADED OPTIONS

- Stocks
- Foreign Currency
- Stock Indices
- Futures

SPECIFICATION OF EXCHANGE-TRADED OPTIONS

- Expiration date
- Strike price
- European or American
- Call or Put (option class)

MARKET MAKERS

- Most exchanges use market makers to facilitate options trading
- A market maker quotes both bid and ask prices when requested
  - with the bid-ask spread within a maximum limit,
  - with the size no less than a minimum requirement,
  - at no less than a certain percentage of time (lower limit)
  - on no less than a certain fraction of securities that they cover.
- The market maker does not know whether the individual requesting the quotes wants to buy or sell
  - The benefit of market making is the bid-ask spread;
  - The risk is market movements.
  - The risk and cost of options market making is relatively large. The bid-ask is wide (stock options). The tick size is 10 cents on options with prices higher than $3. It is 5 cents otherwise.

MARKET MAKERS

- Since there can be hundreds of options underlying one stock, when the stock price moves, quotes on the hundreds of options must be updated simultaneously.
    - Quote message volume is dramatically larger than trade message volume.
    - The risk exposure is large compared to the benefit.
    - When a customer who has private information on the underlying stock (say, going up), the customer can buy all the call options and sell all the put options underlying one stock.
    - The market maker's risk exposure is the sum of all the quote sizes he honors on each contract.
    - Market makers hedge their risk exposures by buying/selling stocks according to their option inventories.
    - Market makers nowadays all have automated systems to update their quotes, and calculate their optimal hedging ratios.
    - Options market makers are no longer individual persons, but are well-capitalized firms.

PROPERTIES OF STOCK OPTIONS

STOCK OPTIONS

FACTORS AFFECTING THE OPTION PRICES

- The value of an option is determined by
  - the current spot (or forward) price (S_t or F_t),
  - the strike price K ,
  - the time to maturity $\tau = T - t$,
  - the option type (Call or put, American or European), and
  - the dynamics of the underlying security (e.g., how volatile the security price is).
- Out-of-the-money options do not have intrinsic value, but they have time value.
- Time value is determined by time to maturity of the option and the dynamics of the underlying security.
- Generically, we can decompose the value of each option into two components:

option value = intrinsic value + time value.

FACTORS AFFECTING OPTION PRICES

- There are six factors affecting the price of a stock option.

	European Call	European Put	American Call	American Put
Stock Price	+	-	+	-
Strike price	-	+	-	+
Maturity	?	?	+	+
Volatility	+	+	+	+
Risk-free rate	+	-	+	-
Dividends	-	+	-	+

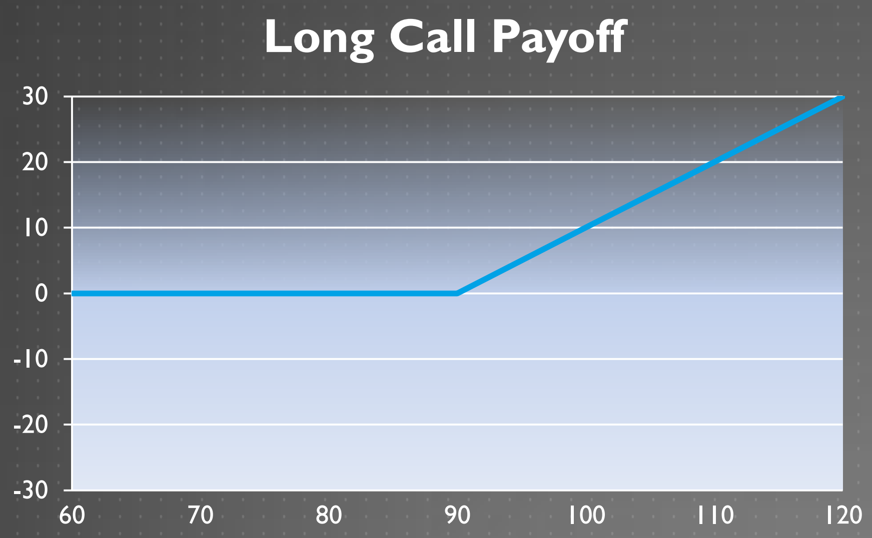
AN EXAMPLE: CALL OPTION ON A STOCK INDEX

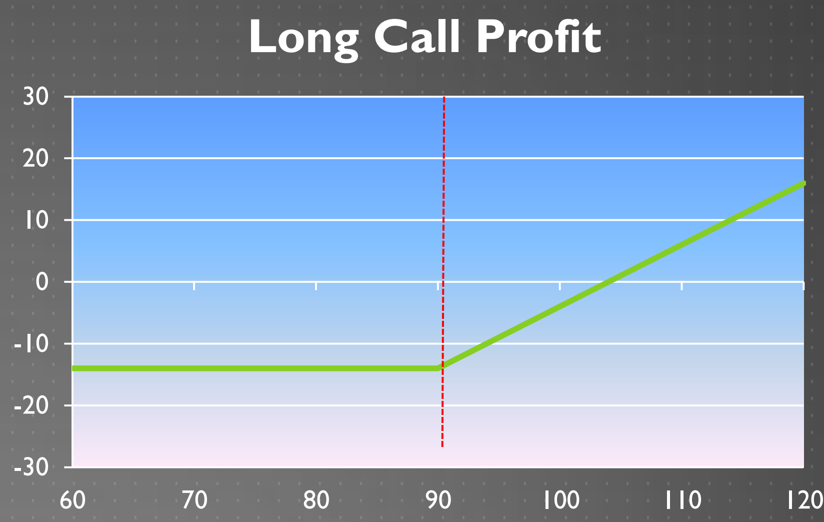
- Consider a European call option on a stock index. The current index level (spot S_t) is 100. The option has a strike (K) of \$90 and a time to maturity (T − t) of 1 year. The option has a current value (c_t) of \$14.

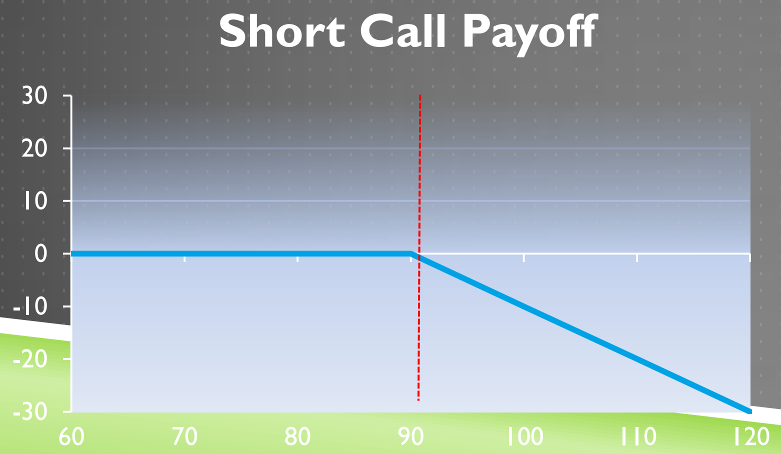
  Assume interest rate is 0%.

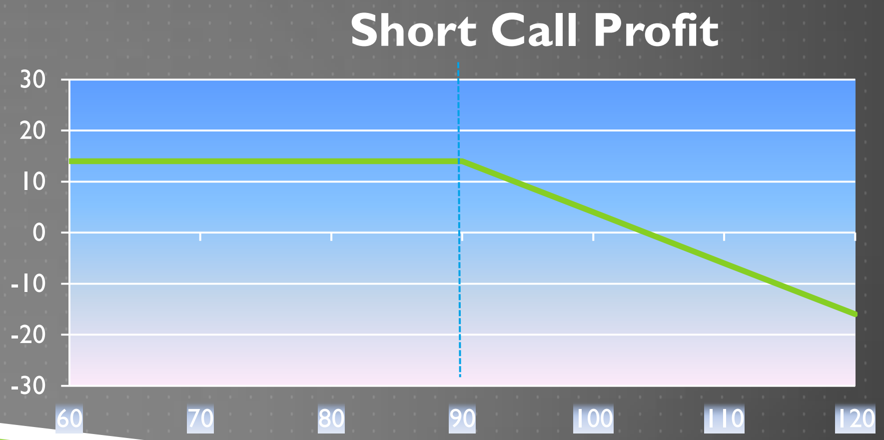
  - Is this option in-the-money or out-of-the-money (wrt to spot)? St > K so in the money
  - What's intrinsic value for this option? What's its time value? 10 and 4
  - If you hold this option, what's your terminal payoff?
    - What's your payoff and P&L if the index level reaches 1000, 900, or 800 at the expiry date T ?
  - If you write this option and have sold it to the exchange, what does your terminal payoff look like?
    - What's your payoff and P&L if the index level reaches 100, 90, or 80 at the expiry date T ?

PAY-OFF AND P&L FOR CALL OPTION









ANOTHER EXAMPLE: PUT OPTION ON AN EXCHANGE RATE

- Consider a European put option on the dollar price of pound (GBPUSD). The current spot exchange rate (S_t) is \$1.6285 per pound. The option has a strike (K) of \$1.61 and a time to maturity ($T - t$) of 1 year. The 1-year forward price ($F_{t,T}$) is \$1.61. The dollar continuously compounding interest rate at 1-year maturity (r_d) is 5%. The option (p_t) is priced at \$0.0489.
  - From the above information, can you infer the continuously compounding interest rate at 1-year maturity on pound (r_f)?
  - Is this option in-the-money or out-of-the-money wrt to spot? What's the moneyness in terms of forward?
  - In terms of forward, what's intrinsic value for this option? What's its time value?
  - If you hold this option, what's your terminal payoff, if the dollar price of pound reaches 1.41, 1.61. or 1.81 at the expiry date T ?

ANOTHER EXAMPLE: PUT OPTION ON AN EXCHANGE RATE

- Review the forward pricing formula:

$$F_T = S_0 e^{(r_d - r_f)(T-t)}$$

$$r_d - r_f = f - s$$

$$r_f = r_d - f + s = 0.05 - \ln\left(\frac{1.61}{1.6285}\right) = 6.14\%$$

  - This is covered interest rate parity: Annualized forward return *f*-*s* on exchange rates equals interest rate differential (rd − rf) between the two currencies.
- Long a put option pays off, (K − ST)+ , and bets on the underlying currency (pound) depreciates.
- Shorting a put option bets on pound appreciates.
- How does it differ from betting using forwards?

UPPER AND LOWER BOUNDS FOR OPTION PRICES (1)

- Assumptions:
  - no transaction costs
  - borrowing and lending at risk-free interest rate.
- Definitions:
  - S: current stock price; S_T: stock price at time *T*;
  - K: strike price;
  - *T*: time of expiration of option; *t*: current time;
  - *r*: risk free interest rate;
  - *C*: value of American call; *c*: value of European call;
  - *P*: value of American put; *p*: value of European put;
  - σ : volatility of stock price.

UPPER AND LOWER BOUNDS FOR OPTION PRICES (2)

- Upper Bounds
- Calls: $c \leq S$ and $C \leq S$
  - no matter what happens, the option can never be worth more than the stock.
- Puts: $p \leq K$ and $P \leq K$
  - the option can never be worth more than K.
  - For European options, the option at time *T* will not be worth more than *K*; it follows that at time *0* $p \leq Ke^{-rT}$.

UPPER AND LOWER BOUNDS FOR OPTION PRICES (3)

- Lower bounds for Europeans calls on non-dividend-paying stocks

- Consider two strategies:
- Strategy *A*: Buy 1 European call option and invest $e^{-rT}K$ in risk-free asset. Cost = $c + e^{-rT}K$.
- Strategy *B*: Buy 1 unit of stock at 0 and hold. Cost = S.
- There are two cases:
- Case 1: $S_T > K$ ==> the call option is exercised and portfolio A is worth S_T just like portfolio B.
- Case 2: $K > S_T$ ==> the call option expires worthless and the portfolio is worth K. Portfolio B is worth S_T.

UPPER AND LOWER BOUNDS FOR OPTION PRICES (4)

- Portfolio A is always worth as much as, or even more than portfolio B. Portfolio A is worth max(S_T,K).
- Hence:
  - $c + Ke^{-rT} > S$
  - $c > S - Ke^{-rT}$.
- Since the worst that can happen to a call option is that it expires worthless, its value must be positive ==> $c > 0$, therefore
  - $c > \max(S - Ke^{-rT}, 0)$.

UPPER AND LOWER BOUNDS FOR OPTION PRICES (5)

- Lower bound for European puts on non-dividend-paying stocks

- Consider two strategies.
- Strategy A: Buy 1 European put option and 1 unit of stock for S. Cost = p + S.
- Strategy B: Invest $e^{-rT}K$ in risk-free asset. Cost = $e^{-rT}K$.
- There are two cases:
- Case 1: $S_T < K$ ==> the put option is exercised and portfolio A is worth K just like portfolio B.
- Case 2: $K < S_T$ ==> the put option expires worthless and portfolio A is worth S_T. Portfolio B is worth K.

UPPER AND LOWER BOUNDS FOR OPTION PRICES (6)

- Portfolio A is always worth as much as, or even more than portfolio B. Portfolio A is worth $\max(S_T, K)$.
- Hence:
  - $p + S > Ke^{-rT}$
  - $p > Ke^{-rT} - S.$
- Since the worst that can happen to a put option is that it expires worthless, its value must be positive ==> $p > 0$, therefore
  - $p > \max(Ke^{-rT} - S, 0).$

AMERICAN AND EUROPEAN CALL: EARLY EXERCISE (1)

- There are no advantages in exercising early.
- Strategy A: Buy 1 American call option and invest $e^{-rT}K$ in risk-free asset. Cost = $C + e^{-rT}K$.
- Strategy B: Buy 1 unit of stock at 0 for S. Cost = S.
- If the call option is exercised at time $t < T$, the value of portfolio A is
  - $S - K + e^{-r(T-t)}K$.
- Portfolio A is always worth less than portfolio B if the call option is exercised prior to maturity.
- If the call option is held to expiration, we already saw that the value of portfolio A is $\max(S_T, K)$ ==> portfolio A is always worth at least as much as portfolio B.
- An American call option on a non-dividend-paying stock is therefore worth the same as the corresponding European call option: $C = c$.

AMERICAN AND EUROPEAN CALL: EARLY EXERCISE (2)

- One reason why a call option should not be exercised early is due to the *insurance* that it provides.

  Once the option has been exercised ==> this insurance vanishes.

- Time value of money.

AMERICAN AND EUROPEAN PUT: EARLY EXERCISE (1)

- It may be optimal to exercise an American option on non-dividend-paying stock early.
- Strategy A: Buy 1 American put option and 1 unit of stock .
- Strategy B: invest $e^{-rT}K$ in risk-free asset.
- If the put option is exercised at time $t < T$, the value of portfolio A is K while portfolio B is worth $e^{-r(T-t)}K$ ==> portfolio A is worth more than portfolio B. If the option is hold to expiration: portfolio A becomes worth $\max(K, S_T)$, while portfolio B is worth K. Portfolio A is more attractive than portfolio B regardless the decision of early exercise.
- A Put option, when held in conjunction with the stock, provides an insurance against the stock price falling below a certain level. However, it may be optimal for an investor to give up the insurance and exercise early

AMERICAN AND EUROPEAN PUT: EARLY EXERCISE (2)

- An American put option is worth more than the corresponding European put option $P > p$.

- We already know that
  - $p > Ke^{-rT} - S$.
- For an American put the stronger condition $P \geq K - S$ must hold since immediate exercise is always possible.

- Time value of money.

PUT-CALL PARITY - EUROPEAN OPTIONS (1)

- Relationship between *p* and *c*.
- Strategy A: Buy 1 European call option and invest $e^{-rT}K$ in risk-free asset. Cost = $c + e^{-rT}K$.
- Strategy B: Buy 1 European put option and 1 unit stock for S. Cost = $p + S$.
- Two cases at T:
- Case 1: $S_T > K$;
  - A: The call option is exercised and portfolio A is worth S_T.
  - B: The put option is not exercised and portfolio B is worth S_T.
- Case 2: $S_T < K$;
  - A: The call option is not exercised and portfolio A is worth K.
  - B: The put option is exercised and portfolio B is worth K.

PUT-CALL PARITY - EUROPEAN OPTIONS (2)

- Both A & B are worth: max(S_T, K).
- By Arbitrage Principle, the two strategies must cost the same since they have the same terminal value.

- $c + e^{-rT}K = p + S \implies c = p + S - e^{-rT}K$ and $p = c + e^{-rT}K - S$.

- The value of a European call with a certain exercise price and exercise date can be deduced from the value of a European put with the same exercise price and date, and *vice versa*.

PUT-CALL PARITY - AMERICAN OPTIONS

- Since $P > p$ and $C = c$, it follows that
  - $P > C + e^{-rT}K - S$ or $C - P < S - e^{-rT}K$.
- Strategy A: Buy 1 European call option and cash (no interest) equal to K.
- Strategy B: Buy 1 American put option and 1 unit of the stock.
- We can show that portfolio A is worth more than portfolio B in all circumstances ==> $c + K > P + S$.
- Combining these equations and knowing that $c = C$, we get
  - $S - K < C - P < S - e^{-rT}K$.

EFFECT OF DIVIDENDS

- Lower bound for European call and put:
  - $c > S - D - e^{-rT}K \qquad p > D - S + e^{-rT}K$
- Put-Call Parity
  - European options: $c + D + e^{-rT}K = p + S$
  - American Options: $S - D - K < C - P < S - e^{-rT}K$.
- Bhattacharya (1983) tested lower bounds:
  - $C > \max(S - K, 0)$, 1.3% violation but usually disappeared the next days ==> traders would not have been able to take advantage. When transaction cost were included ==> no violation.
  - $C > S - D - e^{-rT}K$, 7.6% violation. When transaction cost were included ==> no violation.

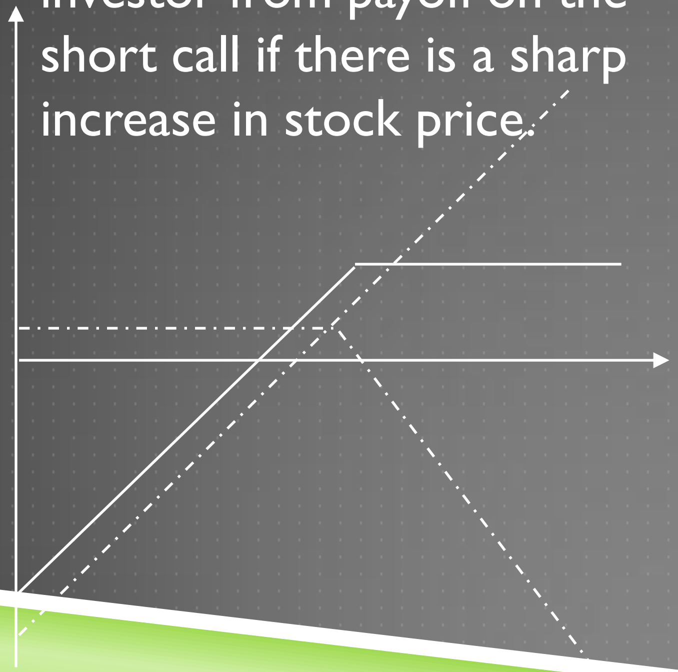
TRADING STRATEGIES INVOLVING OPTIONS

THREE ALTERNATIVE STRATEGIES

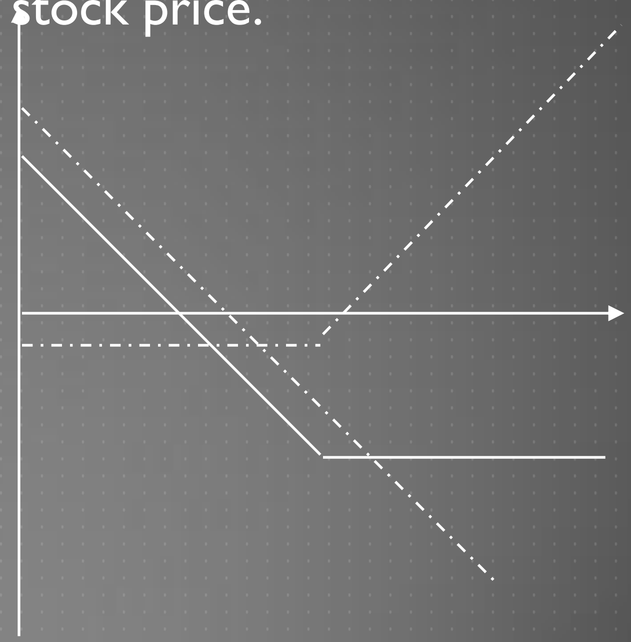
- Take a position in the option and the underlying
- Take a position in 2 or more options of the same type (A spread)
- Combination: Take a position in a mixture of calls & puts (A combination)

TRADING STRATEGIES INVOLVING A SINGLE OPTION AND A STOCK (1)

- *Writing a covered call*: Long in the stock & short call. Long stock position covers the investor from payoff on the short call if there is a sharp increase in stock price.

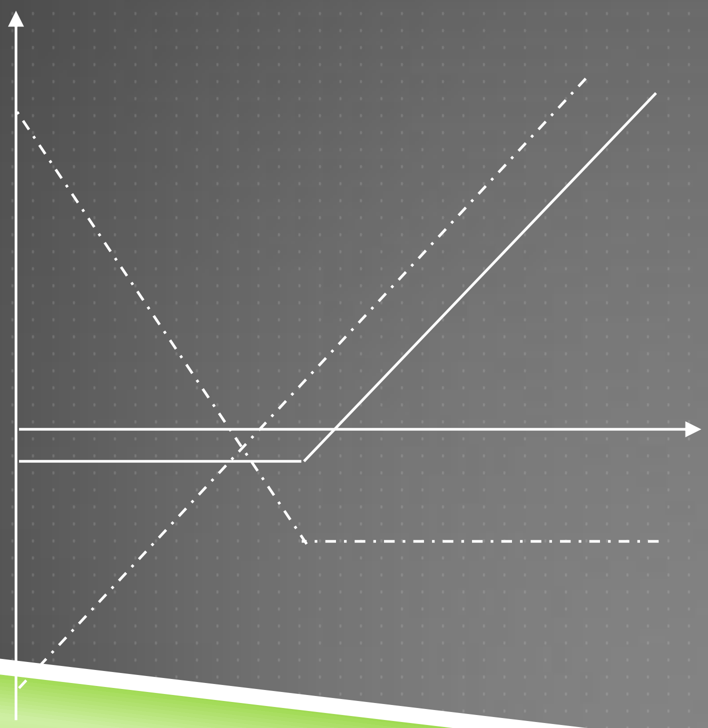


- Reverse of *writing a covered call*: Short in the stock &long call. Long call position covers the investor sharp increase in stock price.

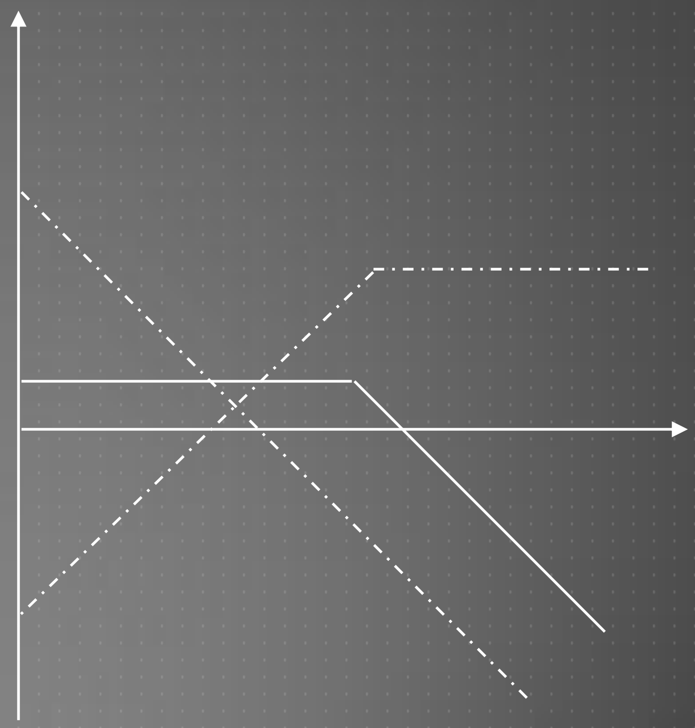


TRADING STRATEGIES INVOLVING A SINGLE OPTION AND A STOCK (2)

- *Protective put*: Long stock & long put.



- Reverse of *protective put*: Short in stock & short put.

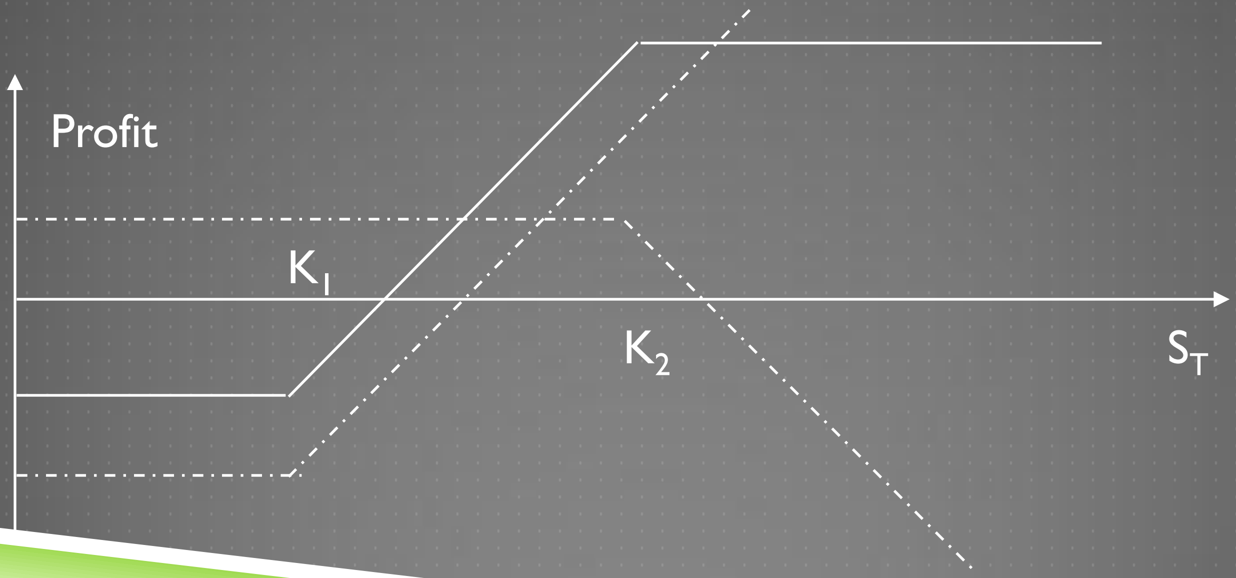


TRADING STRATEGIES INVOLVING A SINGLE OPTION AND A STOCK (3)

- Put-Call Parity:
  - $p + S = c + e^{-rT}K$ ==> $p + S = c + M$
  - $p + S = c + e^{-rT}K$ ==> $S - c = - p + M$
  - where $M = e^{-rT}K$, M is money.
- Long position in a put combined with a long position in stock is equivalent to a long call position plus certain amount of cask.
- Long position in a stock combined with a short position in a call is equivalent to a short put position plus certain amount of cash.

SPREADS - BULL SPREAD (1)

- Spread trading strategy involves two or more options of the same type.
- *Bull Spread*: Buy 1 call option & sell 1 call option with higher strike price. Maturity is the same. Underlying asset is the same. Strike price: $K_1 < K_2$. Assumptions: $E_0(S_T) \uparrow$.



SPREADS - BULL SPREAD (2)

Stock Price	Payoff from long Call	Payoff from short Call	Total Payoff
$S_T \geq K_2$	$S_T - K_1$	$K_2 - S_T$	$K_2 – K_1$
$K_1 < S_T < K_2$	$S_T - K_1$	0	$S_T - K_1$
$S_T \leq K_1$	0	0	0

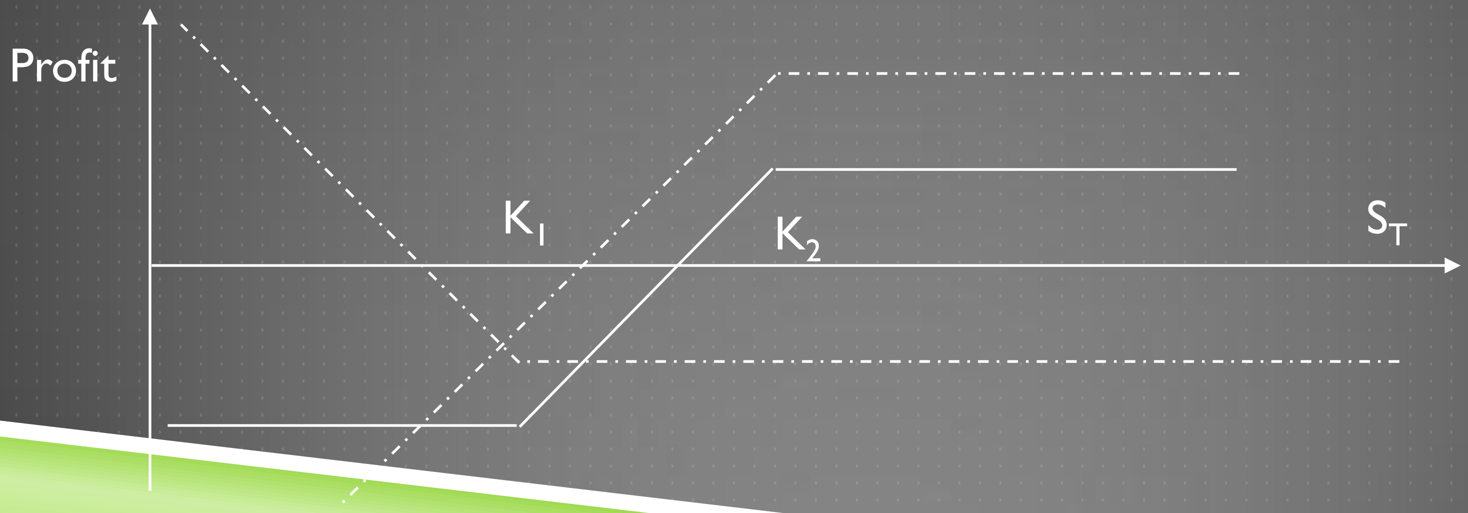
SPREADS - BULL SPREAD (3)

- Bull spread limits both the investor's profits and losses. The investor gives up some of the upside potential profits from the long position and, in return, reduces the initial investment (= difference in price of the two options).

- Three types of bull spreads
  - both option are out of the money (very aggressive strategy, low cost & low probability of high payoff)
  - One call in the money, the other call out of the money
  - Both in the money (very conservative).

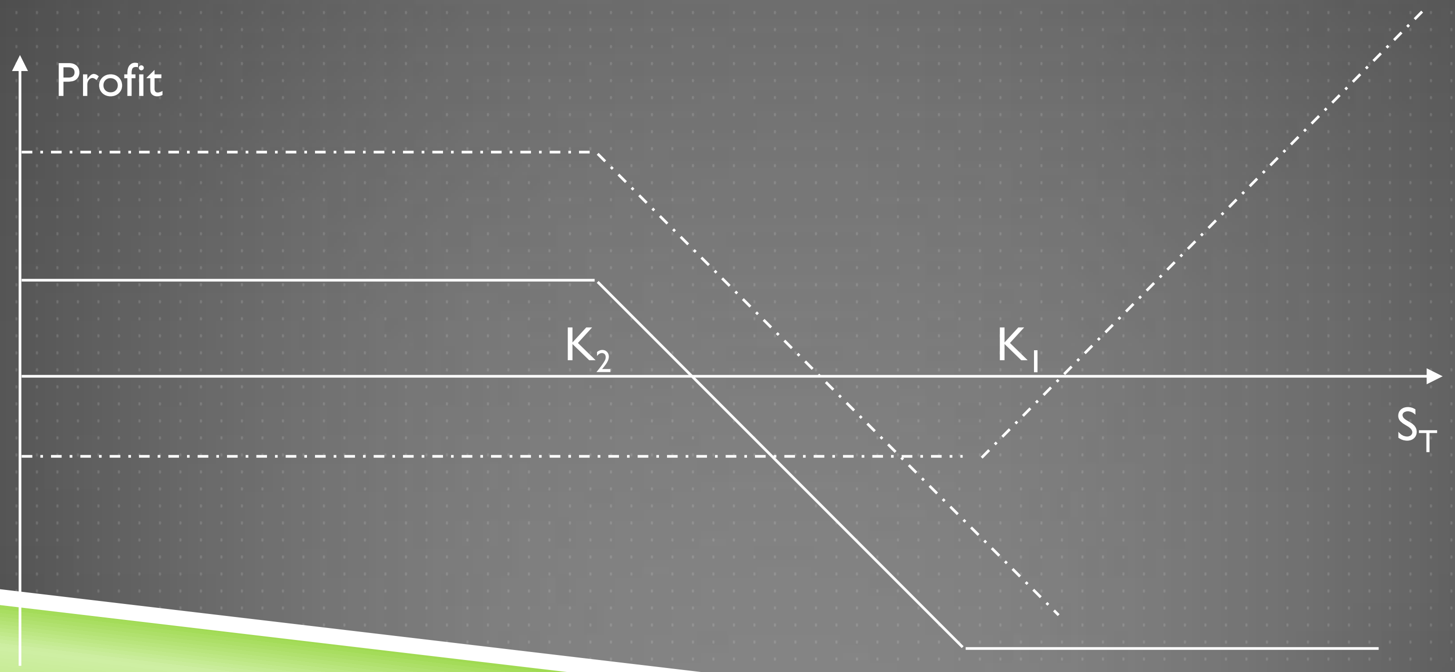
SPREADS - BULL SPREAD (4)

- Bull spread using puts: Buy 1 put option & sell 1 put option with higher strike price. Maturity is the same. Underlying asset is the same. Strike price: $K_1 < K_2$ ==> positive cash flow at time 0 (it was negative with call) & final payoffs are lower than those created using calls.



SPREADS - BEAR SPREAD (1)

- *Bear Spread*: Buy 1 call option & sell 1 call option with lower strike price. Maturity is the same. Underlying asset is the same. Strike price: $K_2 < K_1$. Assumptions: $E_0(S_T) \downarrow$. Positive cash flow at time 0.
- Bear spreads limit both (potential) profits and losses.

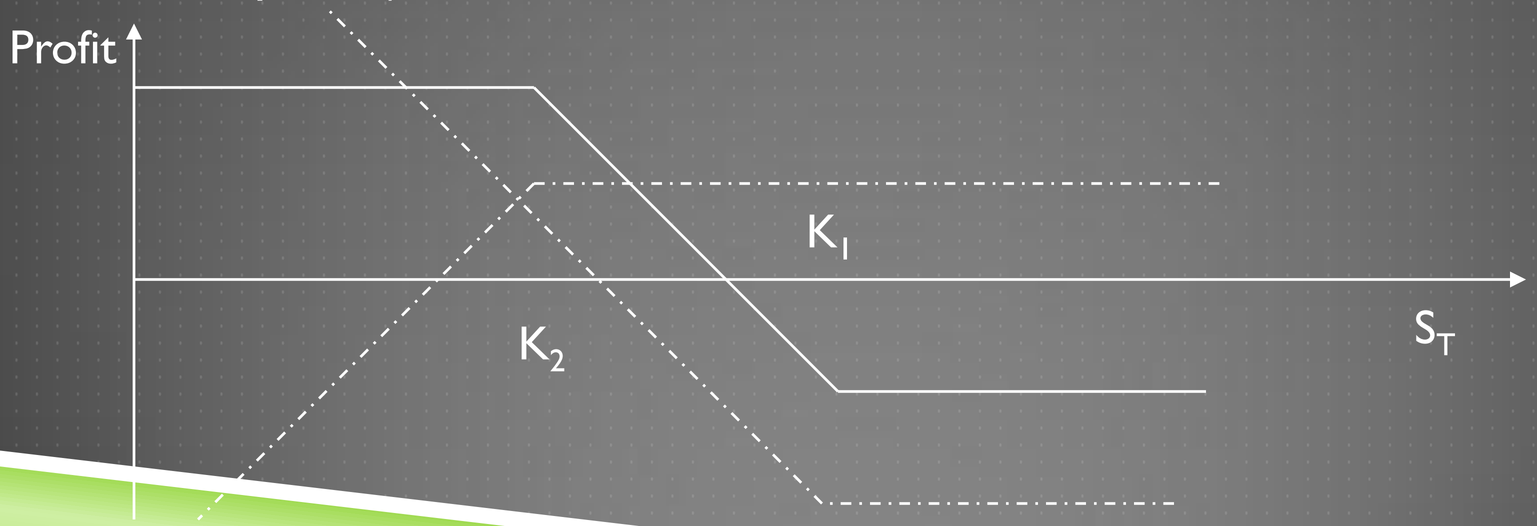


SPREADS - BEAR SPREAD (2)

Stock Price	Payoff from long Call	Payoff from short Call	Total Payoff
$S_T \geq K_1$	$S_T - K_1$	$K_2 - S_T$	$-(K_1 - K_2)$
$K_2 < S_T < K_1$	0	$K_2 - S_T$	$-(S_T - K_2)$
$S_T \leq K_2$	0	0	0

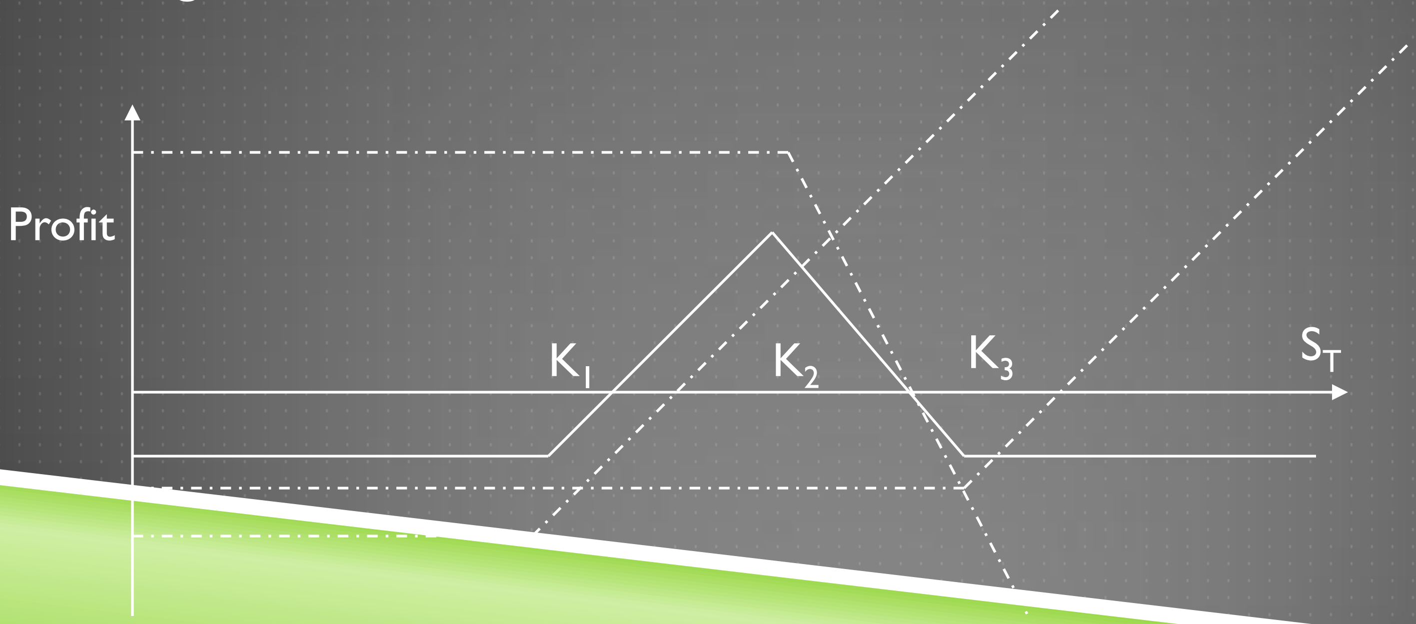
SPREADS - BEAR SPREAD (3)

- Bear spread using puts: Buy 1 put option & sell 1 put option with lower strike price. Maturity is the same. Underlying asset is the same. Strike price: $K_2 < K_1$. Assumptions: $E_0(S_T) \downarrow$. Negative cash flow at time 0. The investor gives up some of the potential profits from the long position and, in return, reduces the initial investment (= difference in price of the two options).



SPREADS - BUTTERFLY SPREAD (1)

- *Butterfly Spread*: Buy 1 call option with low strike price (K_1), buy 1 call option with high strike price (K_3) and sell 2 call options with a strike price (K_2) halfway between K_1 and K_3: generally K_2 is close to the current stock market price. Assumptions: $E_0(S_T) \approx S_0$ (low volatility). Negative cash flow at time 0.



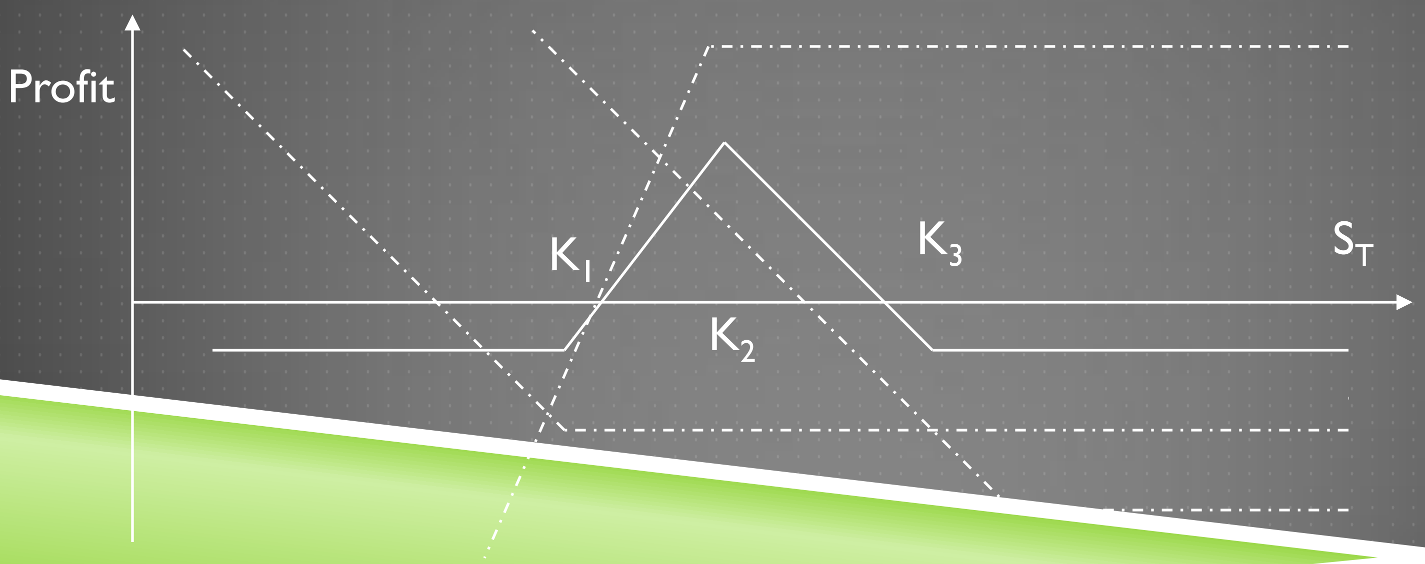
SPREADS - BUTTERFLY SPREAD (2)

- We are assuming that: $K_2 = (K_1 + K_3)/2$.

Stock Price	Payoff 1st call	Payoff 2nd call	Payoff short calls	Total Payoff
$S_T \le K_1$	0	0	0	0
$K_1 < S_T \le K_2$	$S_T - K_1$	0	0	$S_T - K_1$
$K_2 < S_T \le K_3$	$S_T - K_1$	0	$-2(S_T - K_2)$	$K_3 - S_T$
$S_T > K_3$	$S_T - K_1$	$S_T - K_3$	$-2(S_T - K_2)$	0

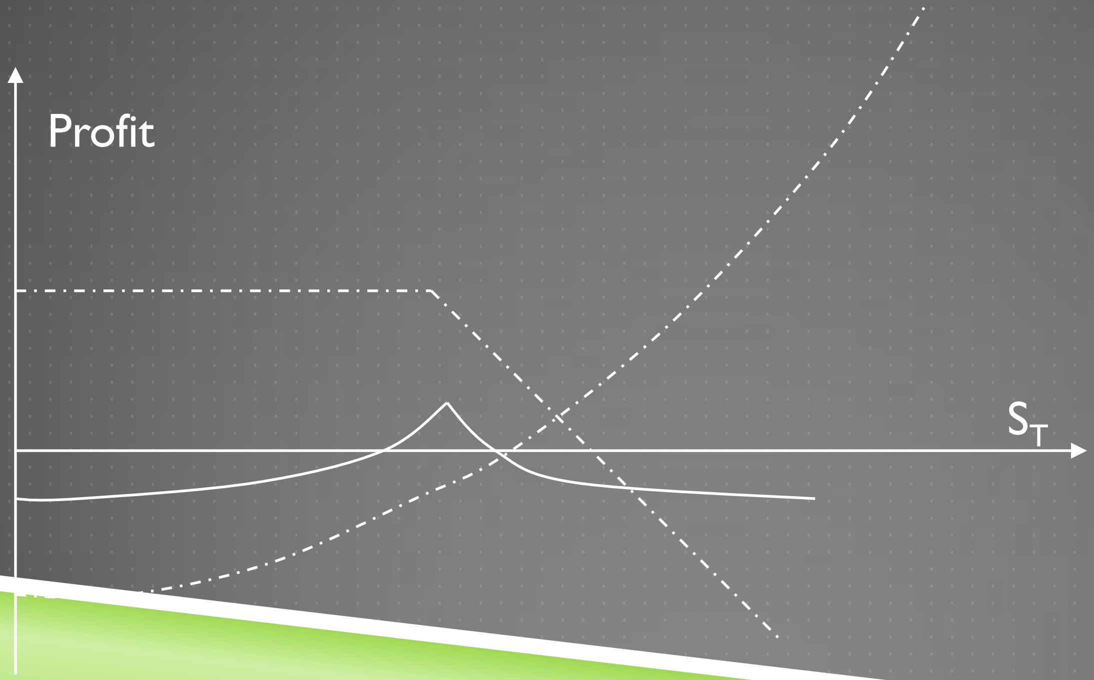
SPREADS - BUTTERFLY SPREAD (3)

- Butterfly Spread using puts: Buy 1 put option with low strike price (K_1), buy 1 put option with high strike price (K_3) and sell 2 put options with a strike price (K_2) halfway between K_1 and K_3.
- Assumptions: $E_0(S_T) \approx S_0$ (low volatility). Negative cash flow at time 0: just like using calls - Put-call parity can be used to show that the initial investment is the same in both cases.



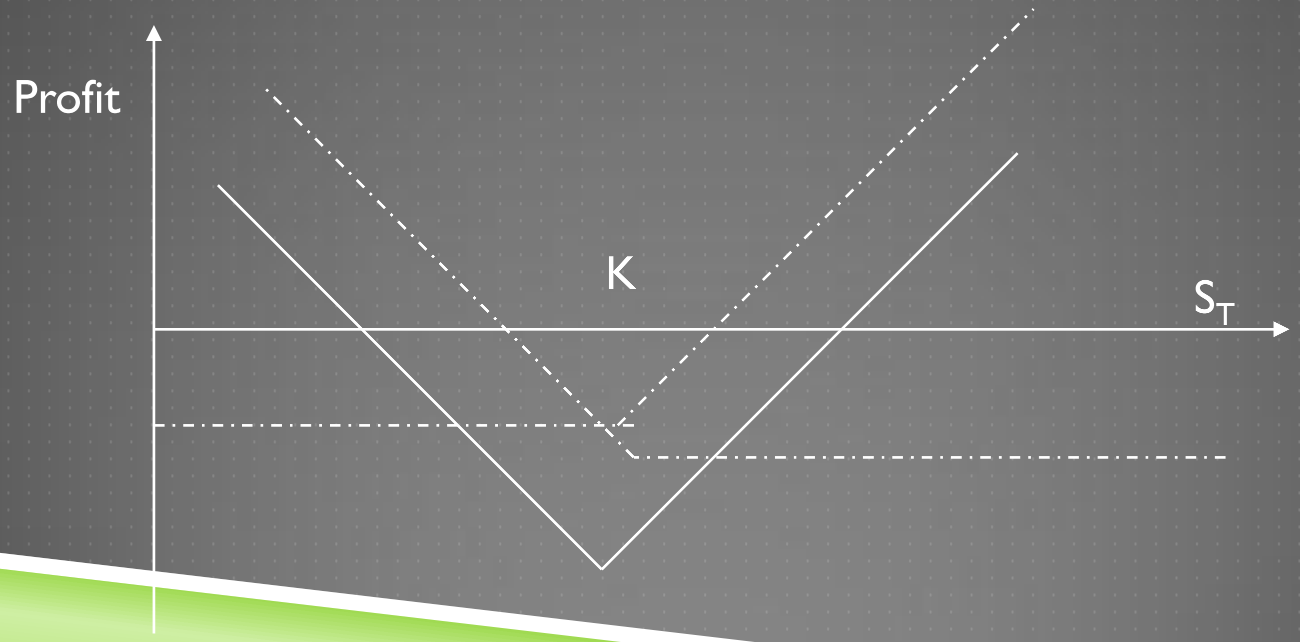
SPREADS - CALENDAR SPREAD

- *Calendar Spread*: Sell 1 call option and buying a longer maturity call option with the same strike price ==> same strike price but different expiration dates.



COMBINATION - STRADDLE (1)

- *Straddle*: Buy 1 call and 1 put. Same strike price and maturity. If the price of the underline asset does NOT move ==> loss. If the price moves ==> profits.



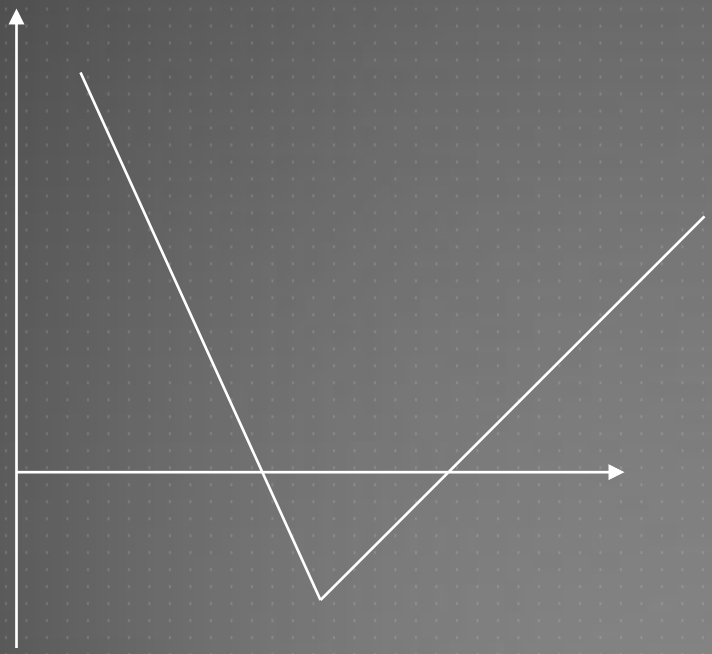
COMBINATION - STRADDLE (2)

- Straddle is a good strategy for stock of a company subject to a takeover bid: If the bid is successful ==> S ↑; if the bid unsuccessful S ↓.

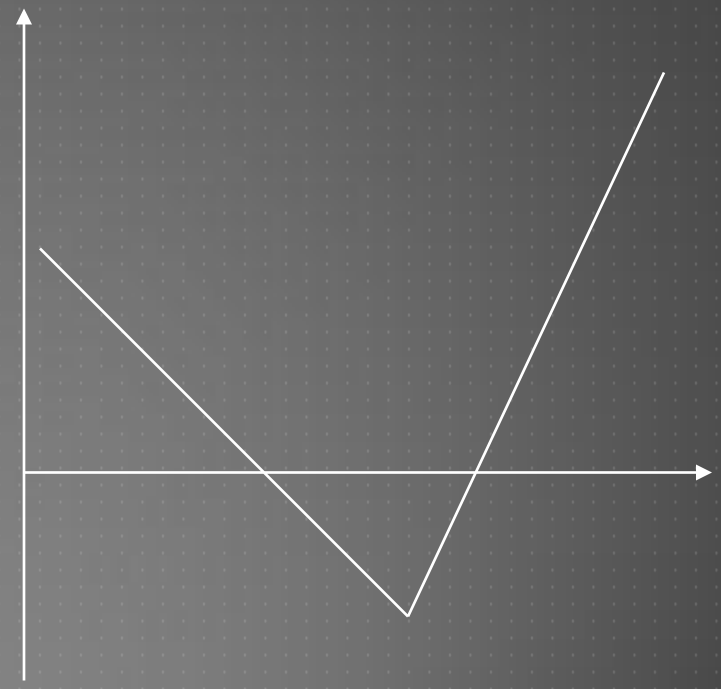
Stock Price	Payoff Call	Payoff Put	Total Payoff
$S_T \leq X$	0	$X - S_T$	$X - S_T$
$S_T > X$	$S_T - X$	0	$S_T - X$

COMBINATION - STRIP & STRAP

- *Strip*: Buy 1 call and 2 puts. Same maturity and strike price.

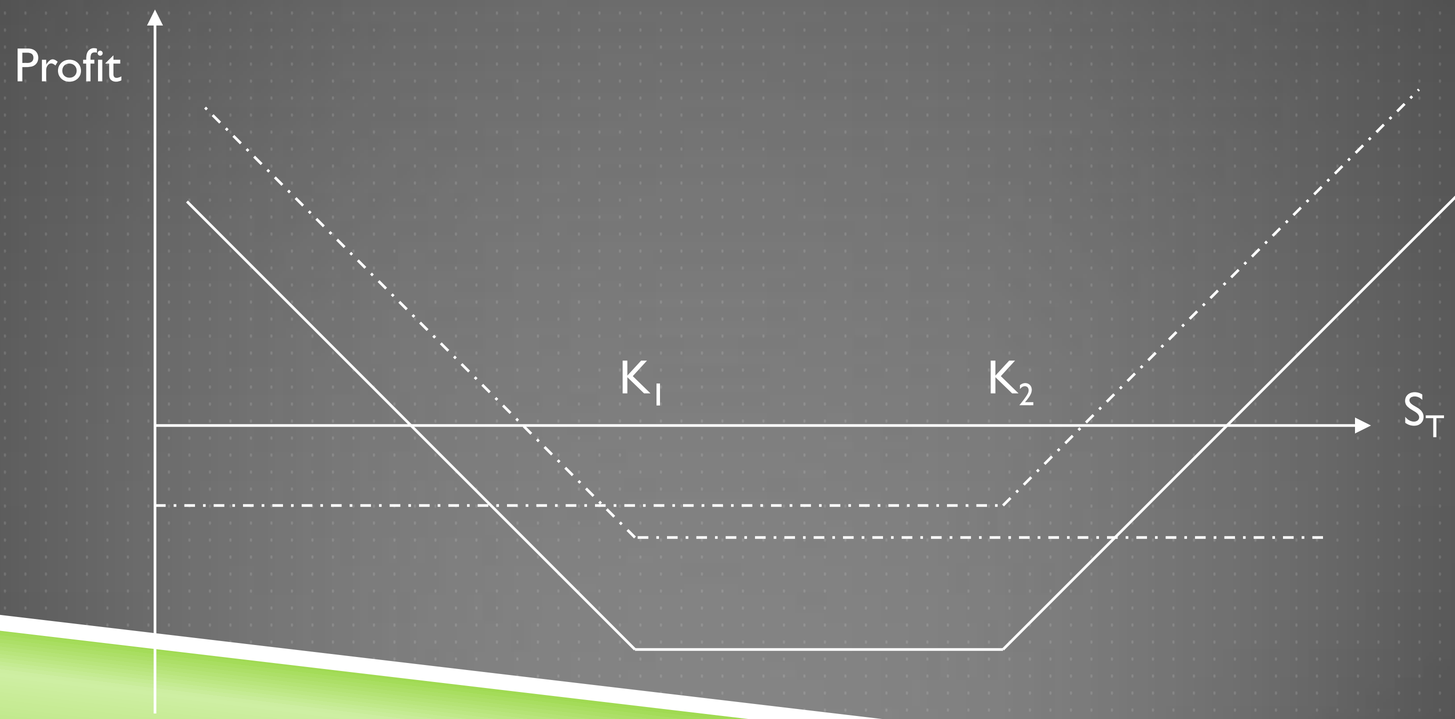


- *Strap*: Buy 2 calls and 1put. Same maturity and strike price.



COMBINATION - STRANGLE (1)

- *Strangle*: Buy 1 put and 1 call with the same maturity but different strike prices.



AN OBSERVATION AND AN INTERESTING QUESTION

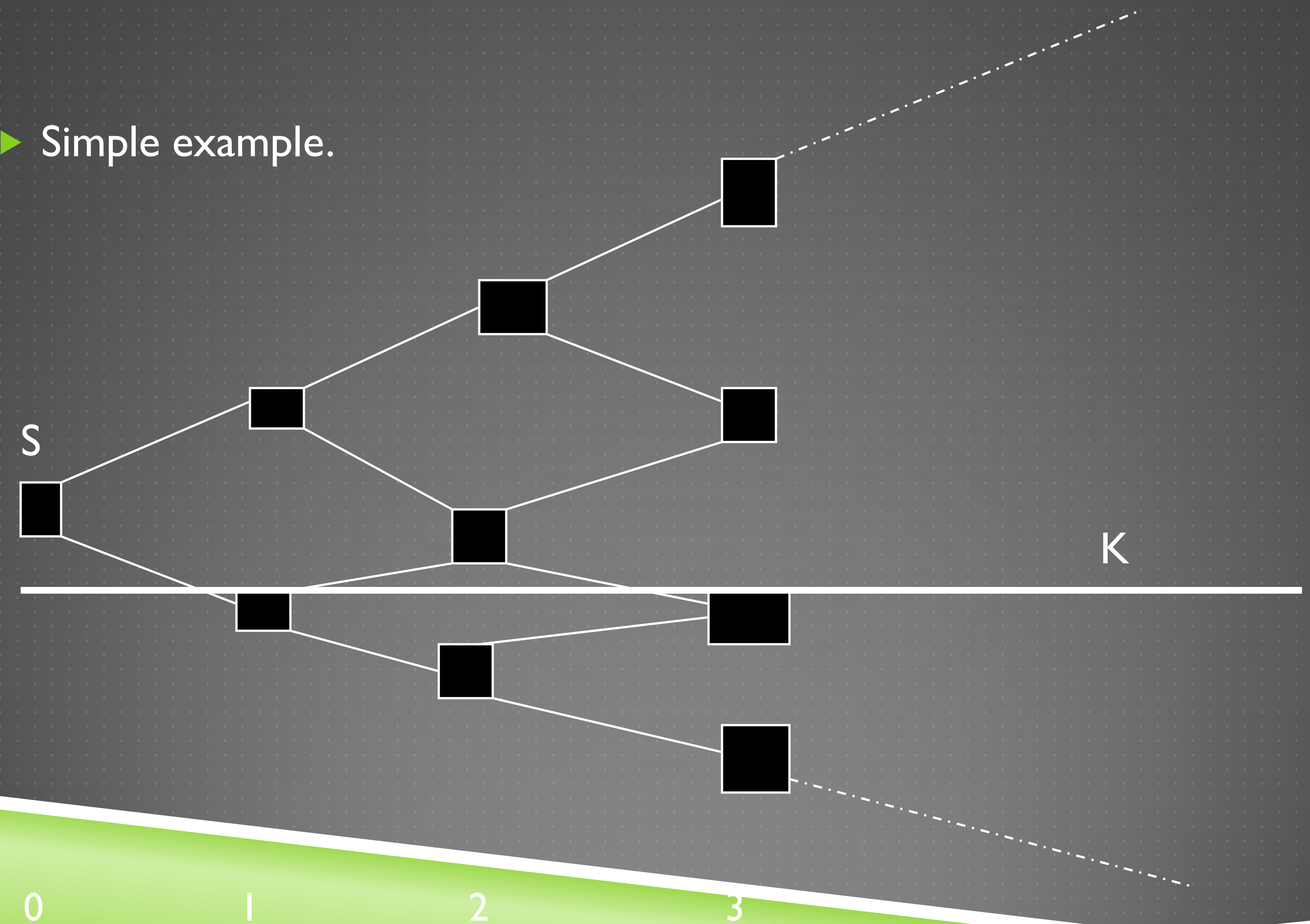
- Observation: Those strategies may be also shorted.

- Interesting Question: Straddle, Strip, Strap & Strangle: What are we really buying and selling?

BINOMIAL TREES

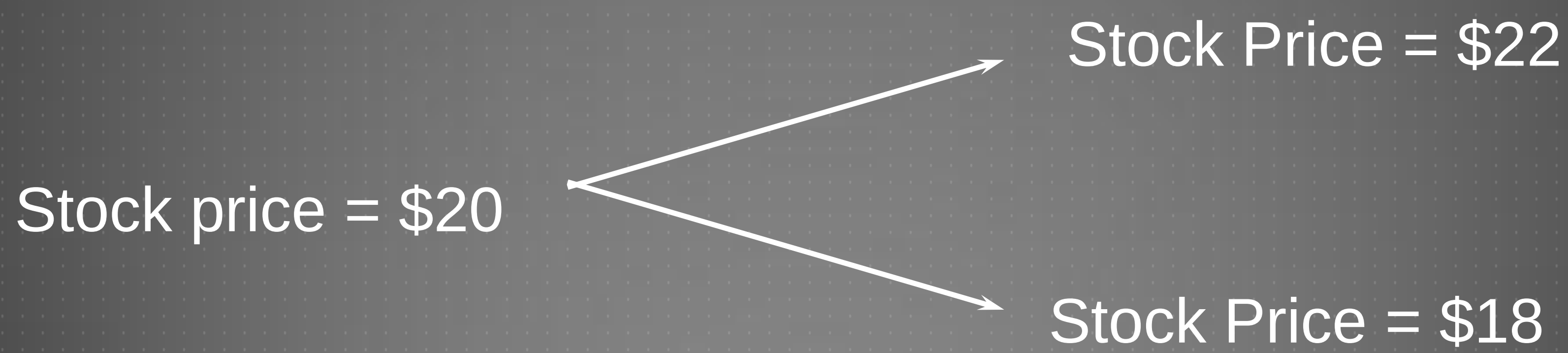
BINOMIAL TREE

- Simple example.



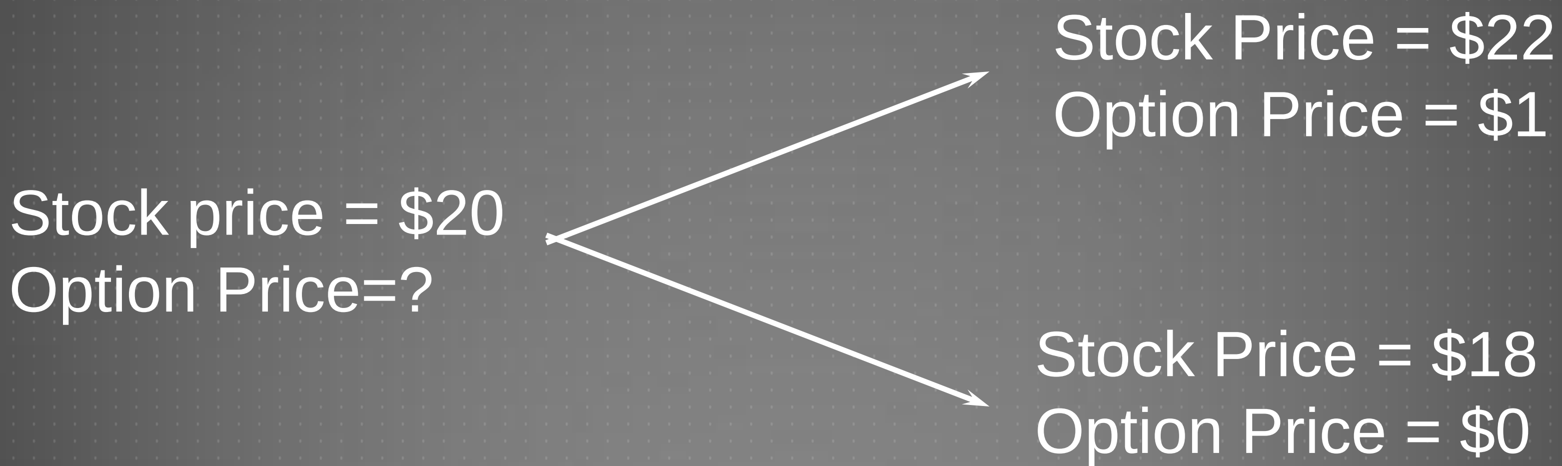
A SIMPLE BINOMIAL MODEL

- A stock price is currently $20
- In three months it will be either $22 or $18



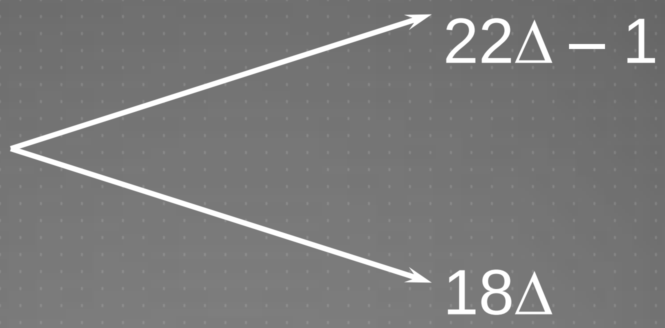
A CALL OPTION

A 3-month call option on the stock has a strike price of 21.



SETTING UP A RISKLESS PORTFOLIO

- Consider the Portfolio: long position in Δ shares of the stock and short 1 call option



- Portfolio is riskless when $22\Delta - 1 = 18\Delta$ or $\Delta = 0.25$

VALUING THE PORTFOLIO (RISK-FREE RATE IS 12%)

- The riskless portfolio is:

  long 0.25 shares
  short 1 call option

- The value of the portfolio in 3 months is
  $22*0.25 - 1 = 4.50 = 18*0.25$
- The value of the portfolio today is
  $4.5e^{-0.12*0.25} = 4.3670$

VALUING THE OPTION

- The portfolio that is

  long 0.25 shares short 1 option

  is worth 4.367 today
- The value of the portfolio today will be

  *20*0.25-f=4.367*

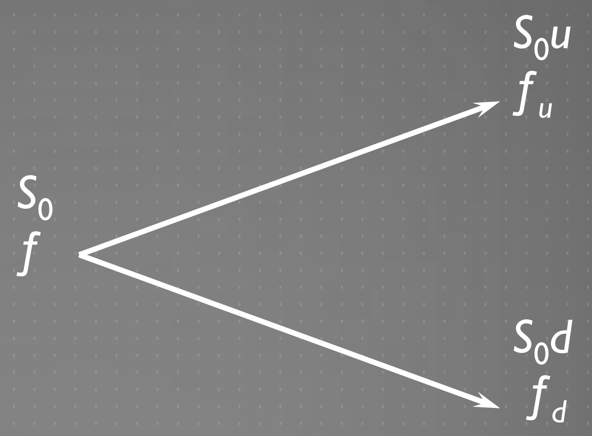
where f represent option price today.

  - The value of the option is therefore *f=5.000 – 4.367*

    f=0.633

GENERALIZATION

- A derivative lasts for time T and is dependent on a stock



GENERALIZATION (CONTINUED)

- Consider the portfolio that is long Δ shares and short 1 derivative

$S_0 - f$

$S_0 u\Delta - f_u$

$S_0 d\Delta - f_d$

- The portfolio is riskless when $S_0 u\Delta - f_u = S_0 d\,\Delta - f_d$ or

$$\Delta = \frac{f_u - f_d}{S_0 u - S_0 d}$$

GENERALIZATION (CONTINUED)

- Value of the portfolio at time T is $S_0 u \Delta - f_u$
- Value of the portfolio today is $(S_0 u \Delta - f_u)e^{-rT}$
- Another expression for the portfolio value today is $S_0\Delta - f$
- Hence

$$f = S_0\Delta - (S_0 u \Delta - f_u)e^{-rT}$$

GENERALIZATION (CONTINUED)

- Substituting for Δ we obtain

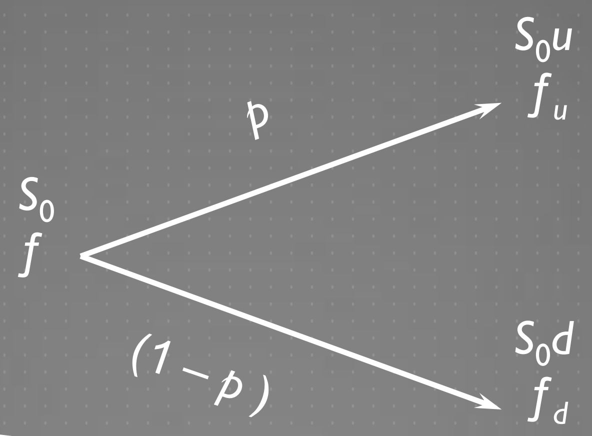
$$f = [\, p\, f_u + (1 - p\,) f_d\,] e^{-rT}$$

where

$$p = \frac{e^{rT} - d}{u - d}$$

RISK-NEUTRAL VALUATION

- $f = [\, p f_u + (1 - p) f_d \,] e^{-rT}$
- The variables p and $(1 - p)$ can be interpreted as the risk-neutral probabilities of up and down movements
- The value of a derivative is its expected payoff in a risk-neutral world discounted at the risk-free rate



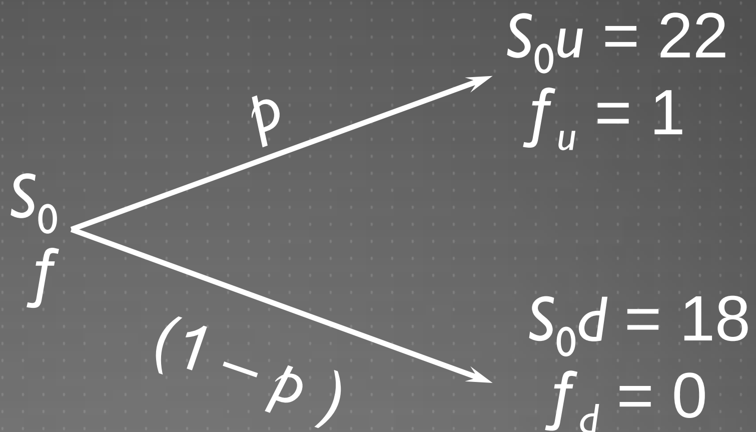
RISK-NEUTRAL VALUATION

- When the probability of an up and down movements are p and 1-p the expected stock price at time T is $S_0 e^{rT}$
- This shows that the stock price earns the risk-free rate
- Binomial trees illustrate the general result that to value a derivative we can assume that the expected return on the underlying asset is the risk-free rate and discount at the risk-free rate
- This is known as using risk-neutral valuation

IRRELEVANCE OF STOCK'S EXPECTED RETURN

- When we are valuing an option in terms of the underlying stock the expected return on the stock is irrelevant
- This is an example of a more general result stating that the expected return on the underlying asset in the real world is irrelevant

ORIGINAL EXAMPLE REVISITED



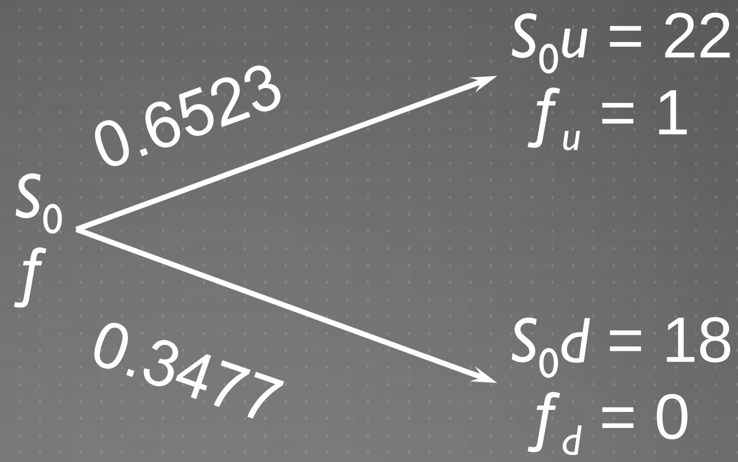
- Since p is a risk-neutral probability

$$20e^{0.12 \times 0.25} = 22p + 18(1 - p); \; p = 0.6523$$

- Alternatively, we can use the formula

$$p = \frac{e^{rT} - d}{u - d} = \frac{e^{0.12 \times 0.25} - 0.9}{1.1 - 0.9} = 0.6523$$

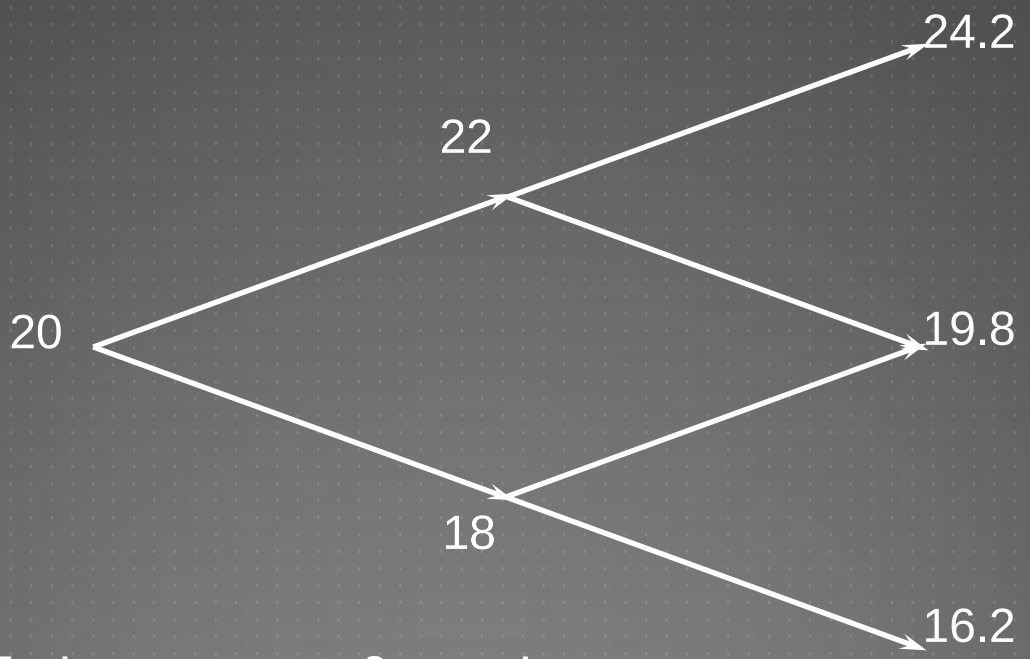
VALUING THE OPTION



The value of the option is

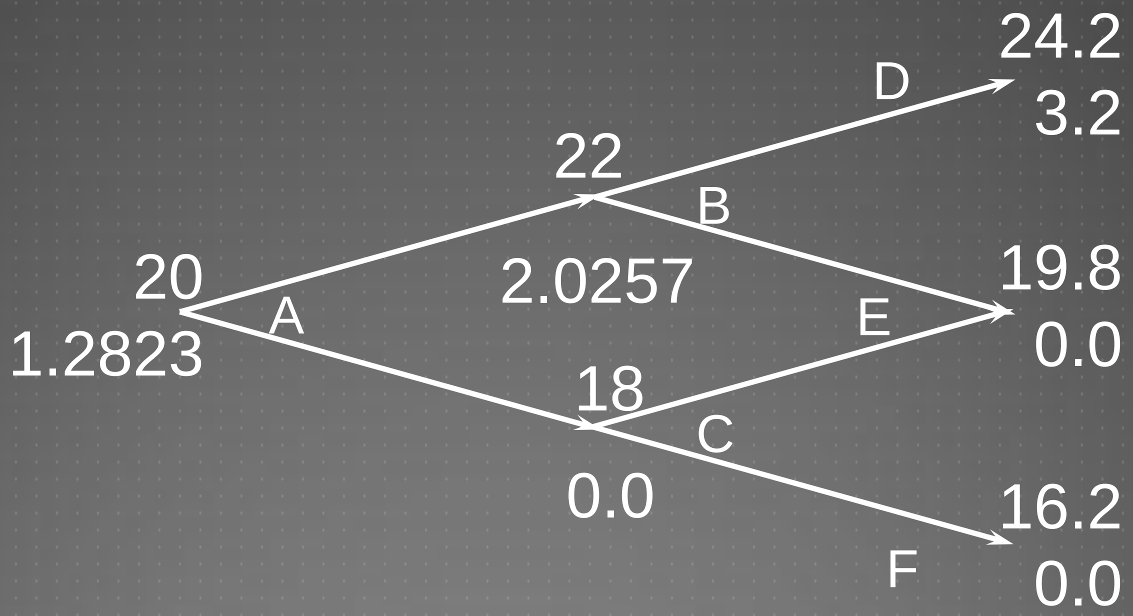
$$e^{-0.12\times0.25}[0.6523\times1 + 0.3477\times0] = 0.633$$

A TWO-STEP EXAMPLE



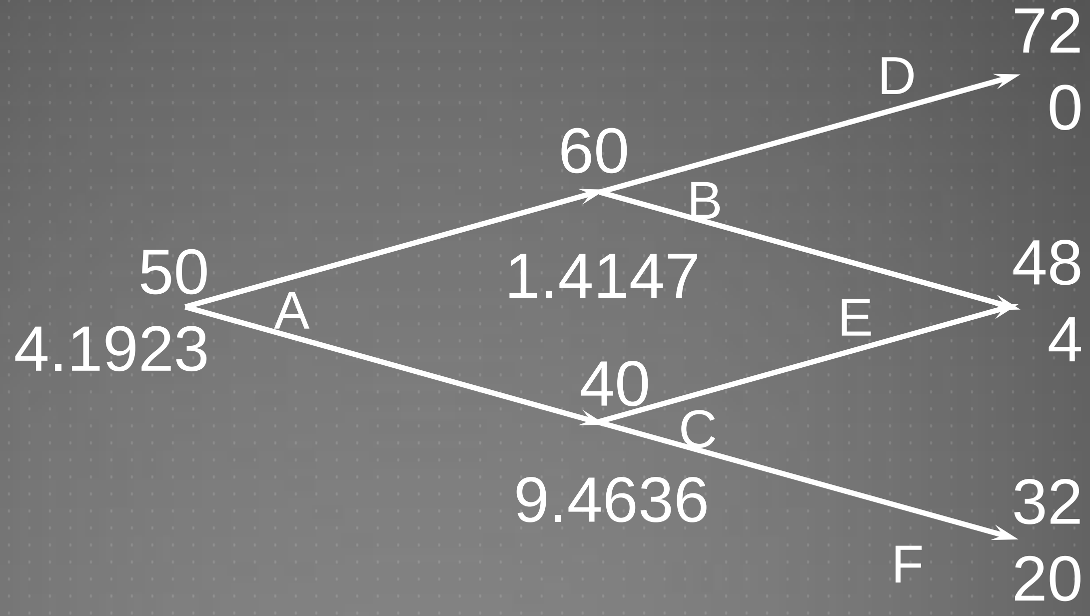
- Each time step is 3 months

VALUING A CALL OPTION



- Value at node B
  $= e^{-0.12\times0.25}(0.6523\times3.2 + 0.3477\times0) = 2.0257$
- Value at node A
  $= e^{-0.12\times0.25}(0.6523\times2.0257 + 0.3477\times0) = 1.2823$

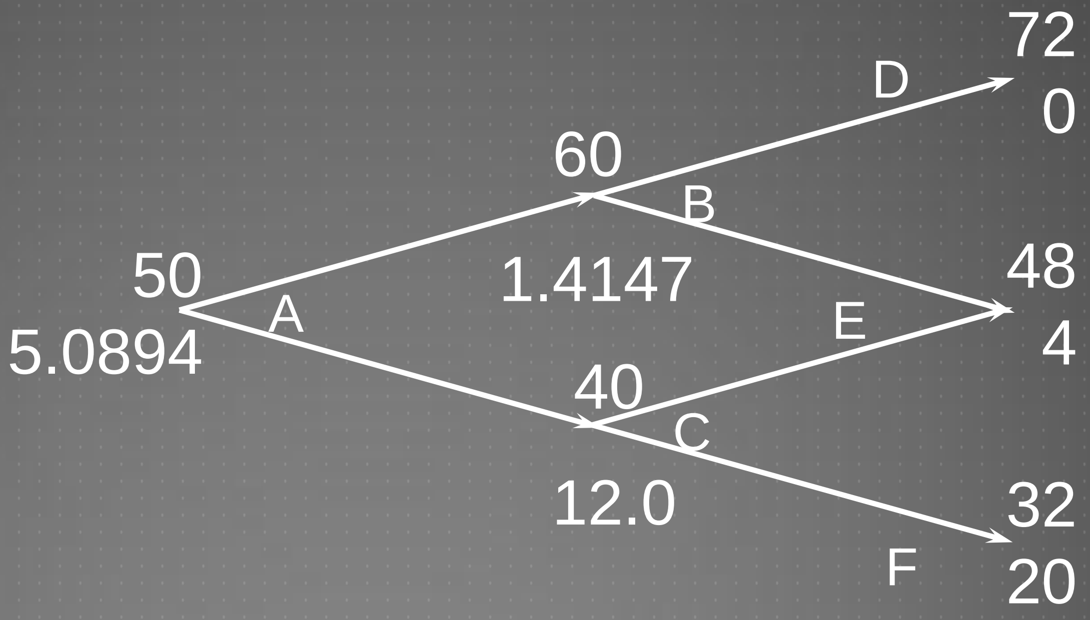
A PUT OPTION EXAMPLE; *K*=52



WHAT HAPPENS WHEN AN OPTION IS AMERICAN

- Work back through the tree from the end to the beginning, testing at each node to see whether early exercise is optimal
- The value of the option at the final node is the same as for the European option
- At earlier nods the value of the option is the greater of
  - The value given by equation 11.5
  - The payoff from early exercise

WHAT HAPPENS WHEN AN OPTION IS AMERICAN



DELTA

- Delta (Δ) is the ratio of the change in the price of a stock option to the change in the price of the underlying stock
- The value of Δ varies from node to node

CHOOSING *U* AND *D*

One way of matching the volatility is to set

$$u = e^{\sigma\sqrt{\delta t}}$$

$$d = e^{-\sigma\sqrt{\delta t}}$$

where σ is the volatility and δt is the length of the time step. This is the approach used by Cox, Ross, and Rubinstein

THE PROBABILITY OF AN UP MOVE

$$p = \frac{a-d}{u-d}$$

- $a = e^{r\Delta t}$ for a nondividend paying stock
- $a = e^{(r-q)\Delta t}$ for a stock index where q is the dividend yield on the index
- $a = e^{(r-r_f)\Delta t}$ for a currency where r_f is the foreign risk-free rate
- $a = 1$ for a futures contract