DERIVATIVE PRICING
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DERIVATIVES

- Derivatives are financial instruments whose returns are derived from those of another financial instrument.
- Cash markets or spot markets
  - The sale is made, the payment is remitted, and the good or security is delivered immediately or shortly thereafter.
- Derivative markets
  - Derivative markets are markets for contractual instruments whose performance depends on the performance of another instrument, the so called underlying.

WAYS DERIVATIVES ARE USED

- To hedge risks
- To speculate (take a view on the future direction of the market)
- To lock in an arbitrage profit
- To change the nature of a liability
- To change the nature of an investment without incurring the costs of selling one portfolio and buying another

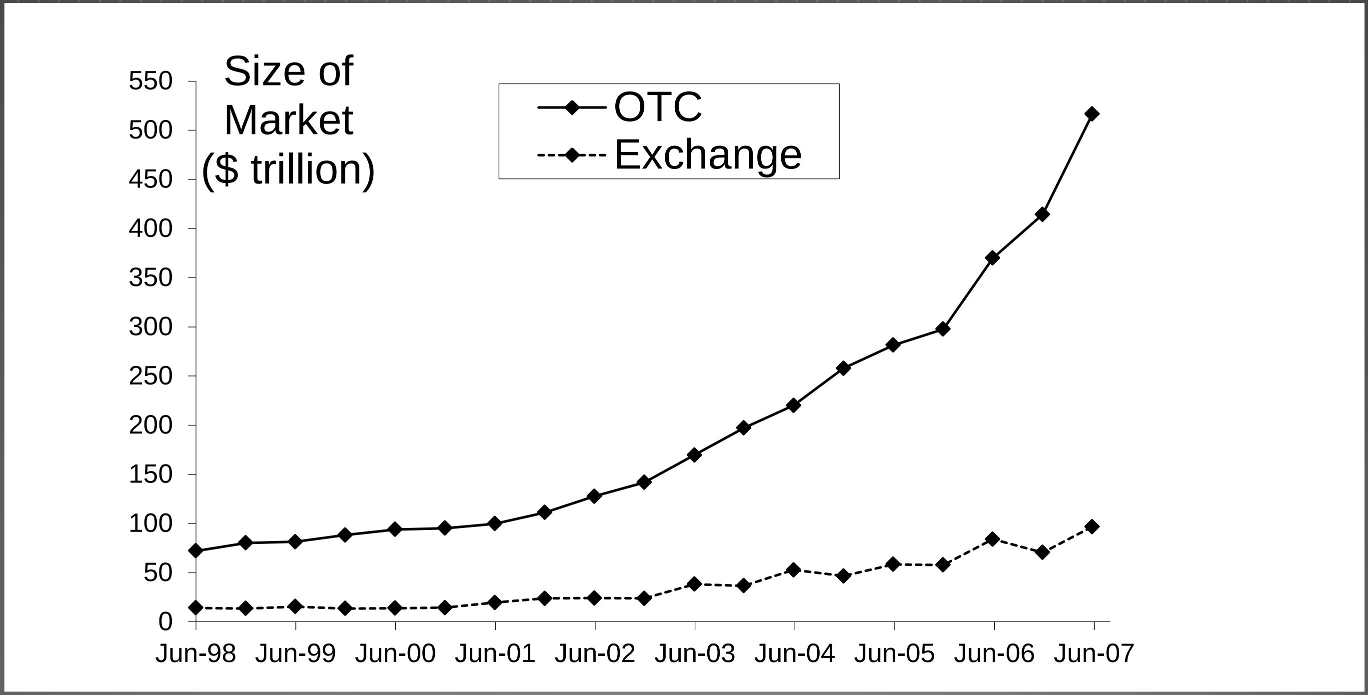
DERIVATIVES MARKETS

- Exchange-traded instruments (Listed products)
  - Exchange traded securities are generally standardized in terms of maturity, underlying notional, settlement procedures ...
  - By the commitment of some market participants to act as market-maker, exchange traded securities are usually very liquid.
    - Market makers are particularly needed in illiquid markets.
  - Many exchange traded derivatives require "margining" to limit counterparty risk.
  - On some exchanges, the counterparty is the exchange itself yielding the advantage of anonymity.

DERIVATIVES MARKETS

- Over-the-counter market (OTC)
  - OTC securities are not listed or traded on an organized exchange.
  - An OTC contract is a private transaction between two parties (counterparty risk).
  - A typical deal in the OTC market is conducted through a telephone or other means of private communication.
  - The terms of an OTC contract are usually negotiated on the basis of an ISDA master agreement (International Swaps and Derivatives Association).

SIZE OF OTC AND EXCHANGE-TRADED MARKETS



Source: Bank for International Settlements. Chart shows total principal amounts for OTC market and value of underlying assets for exchange market

DERIVATIVES PRODUCTS

- Forwards (OTC)
- Futures (exchange listed)
- Swaps (OTC)
- Options (both OTC and exchange listed)

DERIVATIVES PRODUCTS

- Derivatives (or Contingent Claims): A derivative is an instrument whose value depends on the values of other more basic underlying variables

  - Forward/Futures: It is an agreement (contract) to buy/sell an asset at a certain future time for a certain price.
  - Call option: It gives the holder the right to buy the underlying asset by a certain date for a certain price.
  - Put Option: It gives the holder the right to sell the underlying asset by a certain date for a certain price.
  - Swaps: they are agreements between two companies to exchange cash flows in the future according to prearranged formula.

DERIVATIVE TRADERS

- Hedgers
  - to eliminate risk
- Speculators
  - to make money on market expectations
- Arbitrageurs
  - to make money on "markets imperfections"

Some of the largest trading losses in derivatives have occurred because individuals who had a mandate to be hedgers or arbitrageurs switched to being speculators.

REVIEW: VALUATION AND INVESTMENT IN PRIMARY SECURITIES

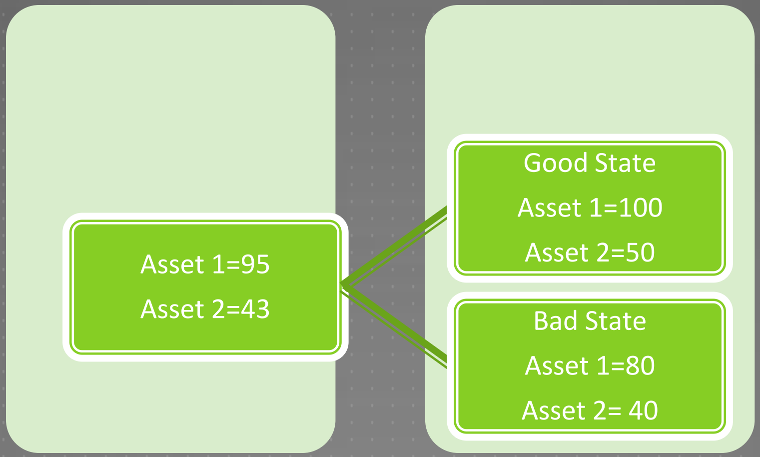
- The securities have direct claims to future cash flows.
- Valuation is based on forecasts of future cash flows and risk:
  - DCF (Discounted Cash Flow Method): Discount forecasted future cash flow with a discount rate that is commensurate with the forecasted risk.
- Investment: Buy if market price is lower than model value; sell otherwise.
- Both valuation and investment depend crucially on forecasts of future cash flows (growth rates) and risks (beta, credit risk).

COMPARE: DERIVATIVE SECURITIES

- Payoffs are linked directly to the price of an "underlying" security.
- Valuation is mostly based on replication/hedging arguments.
  - Find a portfolio that includes the underlying security, and possibly other related derivatives, to replicate the payoff of the target derivative security, or to hedge away the risk in the derivative payoff.
  - Since the hedged portfolio is risk-free, the payoff of the portfolio can be discounted by the risk free rate.
  - Models of this type are called "no-arbitrage" models.
- Key: No forecasts are involved. Valuation is based on cross-sectional comparison.
  - It is not about whether the underlying security price will go up or down (given growth rate or risk forecasts), but about the relative pricing relation between the underlying and the derivatives under all possible scenarios.

ARBITRAGE IN A MICKY MOUSE MODEL

- The current prices of asset 1 and asset 2 are 95 and 43, respectively.
- Tomorrow, one of two states will come true
  - A good state where the prices go up or
  - A bad state where the prices go down



Do you see any possibility to make risk-free money out of this situation?

DCF VERSUS NO-ARBITRAGE PRICING IN THE MICKY MOUSE MODEL

- DCF: Both assets could be over-valued or under-valued, depending on our estimates/forecasts of the probability of the good/bad states, and the discount rate.
- No-arbitrage model: The payoff of asset 1 is twice as much as the payoff of asset 2 in all states, then the price of asset 1 should be twice as much as the price of asset 2.
  - The price of asset 1 is too high relative to the price of asset 2.
  - The price of asset 2 is too low relative to the price of asset 1.
  - I do not care whether both prices are too high or low given forecasted cash flows.
  - Sell asset 1 and buy asset 2, you are guaranteed to make money |arbitrage.
  - Selling asset 1 alone or buying asset 2 alone is not enough.
- Again: DCF focuses on time-series forecasts (of future). No-arbitrage model focuses on cross-sectional comparison (no forecasts)!

FORWARD CONTRACTS

- A forward contract is an agreement to buy or sell an asset at a certain time in the future for a certain price (the delivery price)
- It can be contrasted with a spot contract which is an agreement to buy or sell immediately
- It is traded in the OTC market
- The forward price for a contract is the delivery price that would be applicable to the contract if were negotiated today (i.e., it is the delivery price that would make the contract worth exactly zero)
- The forward price may be different for contracts of different maturities

FORWARD CONTRACTS

- A forward contract is an OTC agreement between two parties to exchange
- an underlying asset
  - for an agreed upon price (the forward price)
  - at a given point in time in the future (the expiry date)
- Example: On July 20, 2007, Party A signs a forward contract with Party B to sell 1 million British pound (GBP) at 2.0489 USD per 1 GBP six month later.
  - Today (July 20, 2007), sign a contract, shake hands. No money changes hands.
  - January 20, 2008 (the expiry date), Party A pays 1 million GBP to Party B, and receives 2.0489 million USD from Party B in return.
  - Currently (July 20), the spot price for the pound (the spot exchange rate) is 2.0562. Six month later (January 20,2008), the exchange rate can be anything (unknown).
  - 2.0489 is the forward price.

FOREIGN EXCHANGE QUOTES FOR GBP, JULY 20, 2007

	Bid	Offer
Spot	2.0558	2.0562
1-month forward	2.0547	2.0552
3-month forward	2.0526	2.0531
6-month forward	2.0483	2.0489

FORWARD CONTRACTS

- The forward prices are different at different maturities.
  - Maturity or time-to-maturity refers to the length of time between now and expiry date (1m, 2m, 3m etc).
  - Expiry (date) refers to the date on which the contract expires.
  - Notation: Forward price $F(t;T)$: t: today, T: expiry, $\tau = T - t$: time to maturity.
  - The spot price $S(t) = F(t; t)$.
- Forward contracts are the most popular in currency and interest rates.

FORWARD PRICE REVISITED

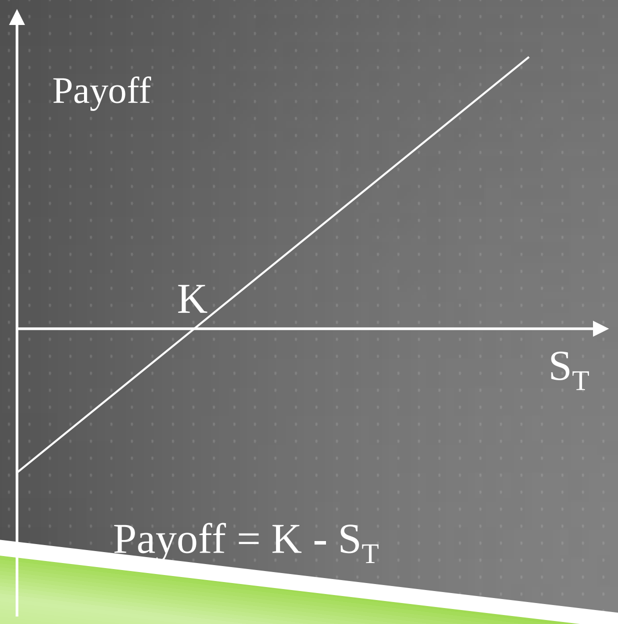
- The forward price for a contract is the delivery price (K) that would be applicable to the contract if were negotiated today. It is the delivery price that would make the contract worth exactly zero.
  - Example: Party A agrees to sell to Party B 1 million GBP at the price of 2.0489USD per GBP six month later
- The party that has agreed to buy has what is termed a long position. The party that has agreed to sell has what is termed a short position.
  - In the previous example, Party A entered a short position and Party B entered a long position on GBP.
  - But since it is on exchange rates, you can also say: Party A entered a long position and Party B entered a short position on USD.

PROFIT AND LOSS (P&L) IN FORWARD INVESTMENTS

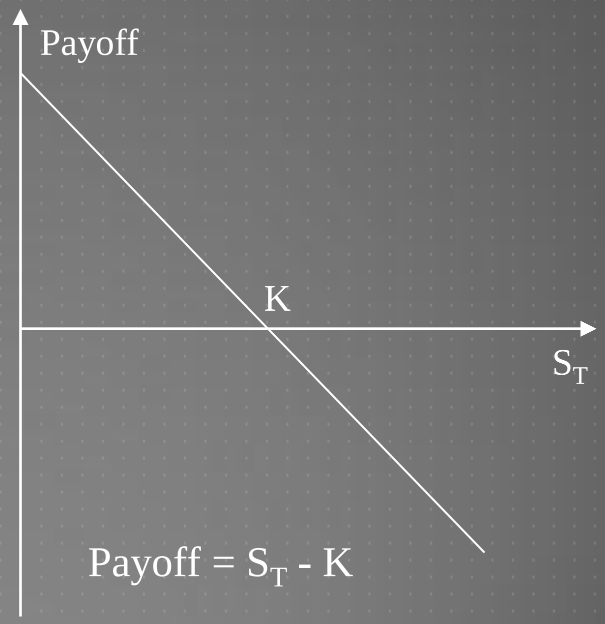
- By signing a forward contract, one can lock in a price ex ante for buying or selling a security.
- Ex post, whether one gains or loses from signing the contract depends on the spot price at expiry.
- In the previous example, Party A agrees to sell 1 million pound at $2.0489 per GBP at expiry. If the spot price is $2 at expiry, what's the P&L for party A?
  - On January 20, 2008, Party A can buy 1 million pound from the market at the spot price of $2 and sell it to Party B per forward contract agreement at $2.0485.
  - The net P&L at expiry is the difference between the strike price (K = 2.0485) and the spot price (S_T = 2), multiplied by the notional (1 million). Hence, $48,500.
- If the spot rate is $2.1 on January 20,2008, what will be the P&L for Party A?
- What's the P&L for Party B?
- Credit risk: There is a small possibility that either side can default on the contract. That's why forward contracts are mainly between big institutions.

PAYOFFS FROM FORWARD CONTRACTS

- LONG POSITION
- K: delivery price
- S_T: price of asset at maturity



- SHORT POSITION
- K: delivery price
- S_T: price of asset at maturity



PAYOFF FROM CASH MARKETS (SPOT CONTRACTS)

- If you buy a stock today (t), what does the payoff function of the stock look like at time T?
  - The stock does not pay dividend.
  - The stock pays dividends that have a present value of D_t .
- What does the time-T payoff look like if you short sell the stock at time t?
- If you buy (short sell) 1 million GBP today, what's your aggregate dollar payoff at time T?
- If you buy (sell) a K dollar par zero-coupon bond with an interest rate of *r* at time *t*, how much do you pay (receive) today? How much do you receive (pay) at expiry T?

PAYOFF FROM CASH MARKETS (SPOT CONTRACTS)

- If you buy a stock today (t), the time-t payoff (T) is
  - S_T if the stock does not pay dividend.
  - $S_T + D_t e^{r(T-t)}$ if the stock pays dividends during the time period *[t,T]* that has a present value of D_t . In this case, $D_t e^{r(T-t)}$ represents the value of the dividends at time T.
- The payoff of short is just the negative of the payoff from the long position:
- $-S_T$ without dividend and $-S_T - D_t e^{r(T-t)}$ with dividend.
  - If you borrow stock (chicken) from somebody, you need to return both the stock and the dividends (eggs) you receive in between.
- If you buy 1 million GBP today, your aggregate dollar payoff at time T is the selling price S_T plus the pound interest you make during the

  time period[t,T]: $S_T e^{r^*(T-t)}$million.
- The zero bond price is the present value of K: $Ke^{r(T-t)}$. The payoff is K for long position and -K for short position.

FUTURES VERSUS SPOT

- Easier to go short: with futures it is equally easy to go short or long.
- A short seller using the spot market must wait for an uptick before initiating a position (the rule is changing...).
- Lower transaction cost.
  - Fund managers who want to reduce or increase market exposure, usually do it by selling the equivalent amount of stock index futures rather than selling stocks.
  - Underwriters of corporate bond issues bear some risk because market interest rates can change the value of the bonds while they remain in inventory prior to final sale: Futures can be used to hedge market interest movements.
  - Fixed income portfolio managers use futures to make duration adjustments without actually buying and selling the bonds.

FUTURES VERSUS FORWARDS

- Futures contracts are similar to forwards, but
- Buyer and seller negotiate indirectly, through the exchange.
- Default risk is borne by the exchange clearinghouse
- Positions can be easily reversed at any time before expiration
- Value is marked to market daily.
- Standardization: quality; quantity; Time.
  - The short position has often different delivery options; good because it reduces the risk of squeezes, bad ... because the contract is more difficult to price (need to price the “cheapest-to-deliver").
- The different execution details also lead to pricing differences,e.g., effect of marking to market on interest calculation.

FUTURES VERSUS FORWARDS

- Futures markets perform the risk transfer function of forward contracts but are more liquid and substantially reduce performance risk.

- Futures markets separate the *marketing* and the *purchasing* decisions (who to sell to or buy from) from the price *insurance* function of forward markets.

- As a consequence, futures markets are also useful even when marketing or purchasing do not arise - e.g. in portfolio management.

FUTURES VERSUS FORWARDS: STANDARDISATION

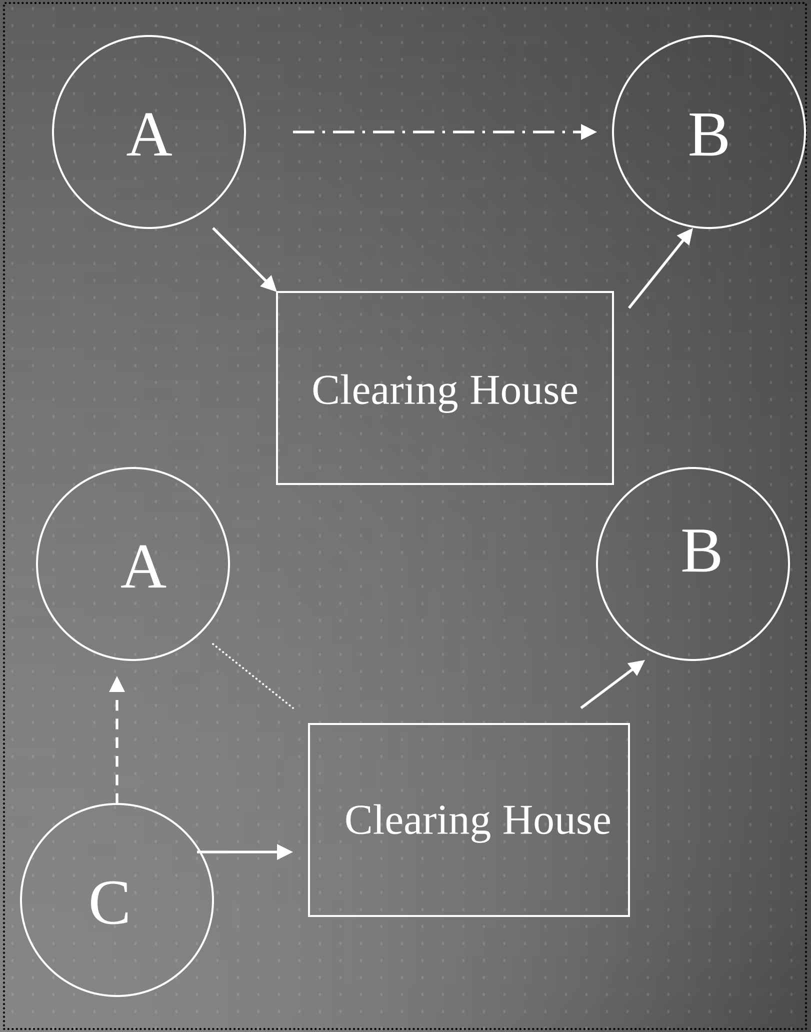
- Standardisation concentrates trading and hence liquidity in a small number of contracts.
- Standardisation by
  - grade - Chicago wheat, “No 2 Red”
  - location: Chicago Points
  - date: Jan, Mar, May, Jul, Sep, Nov.
- Liquidity is important because traders
  - need to be able to trade in size at screen-quoted prices without slippage
  - need to be sure that they can close out positions, since delivery seldom intended.

FUTURES VERSUS FORWARDS

FORWARDS	FUTURES
Private contract between 2 parties	Exchange traded
Non-standard contract	Standard contract
Usually 1 specified delivery date	Range of delivery dates
Settled at end of contract	Settled daily
Delivery or final cash settlement usually occurs	prior to maturity
Some credit risk	Virtually no credit risk

THE CLEARING HOUSE

- The exchange clearing house intermediates all futures transactions. The credit status of the counterparty became irrelevant and contracts became *fungible*. A transactor needs only worry about the credit status of the clearing house (fine in London, N.Y. and Chicago).
- Here A's contract with B is replaced by a contract with the clearing house. If C sells to A, closing out A's short position, B is uninvolved.



MARKETING TO MARKET

- Marketing to market ensures that futures contracts always have zero value - hence the Clearing House does not face any risk. Marketing to market takes place through *margin payments*.
- At the inception of the contract, each party pays initial margin (typically 10% of value contracted) to a margin account held by his broker. Initial margin may be paid in interest-bearing securities (t-bills) so there is no interest cost.
- If futures price rises (falls), the longs have made a paper profit (loss) and the shorts a paper loss (profit). The broker pays losses from and receives any profits into the parties' margin accounts on the morning following trading.
- Loss-making parties are required to restore their margin accounts to the required level during the course of the same day by payment of variation margins in cash; margin in excess of the required level may be withdrawn by profit-making parties.

MARGIN EXAMPLE

- 19 Feb: John Smith sells 10 June FT-SE 100 Index Futures Contracts on LIFFE @ 3749.5. Transaction value is £25 x 3749.5 = $93,737.5. He deposits initial margin of £9,374.
- 26 Feb: FT-SE falls to 3600. Smith's margin account is credited £25 x (3749.5 - 3600) = £3,737.5. He withdraws this money leaving initial margin intact.
- 4 Mar: FT-SE rises to 3800. Smith's margin account is debited £25 x (3800 - 3600) = £5,000. He deposits this sum of money to restore initial margin.
- 11 Mar FT-SE falls to 3700. Smith's margin account credited £25 x (3700 - 3800) = £2,500 to stand at £11,874. He buys 10 June contracts to close his position and withdraws his margin.
- Profit = £11,874 + £3,737.5 - £5,000 - £9,374 = £1,237.5 = £25 x (3749.5 - 3700).

CONTRACT STRUCTURE

- Standard futures markets trade only a small number of contracts per year.
- This concentrates liquidity and gives good execution. However, it implies that contract maturity dates are unlikely to coincide exactly with desired hedge dates.
- Example - LCE (LIFFE) Cocoa trades Mar, May, Jul, Sep & Dec with contracts maturing on 3rd Friday of delivery month.
- This gives rise to basis (structure) risk since the term of the contract and the term of the hedge differ.
- Liquidity concentrates in nearby contracts - longer hedges need to be *rolled*.
- Markets typically exhibit a “roll” over a period of 3-5 days at the start of the delivery month.

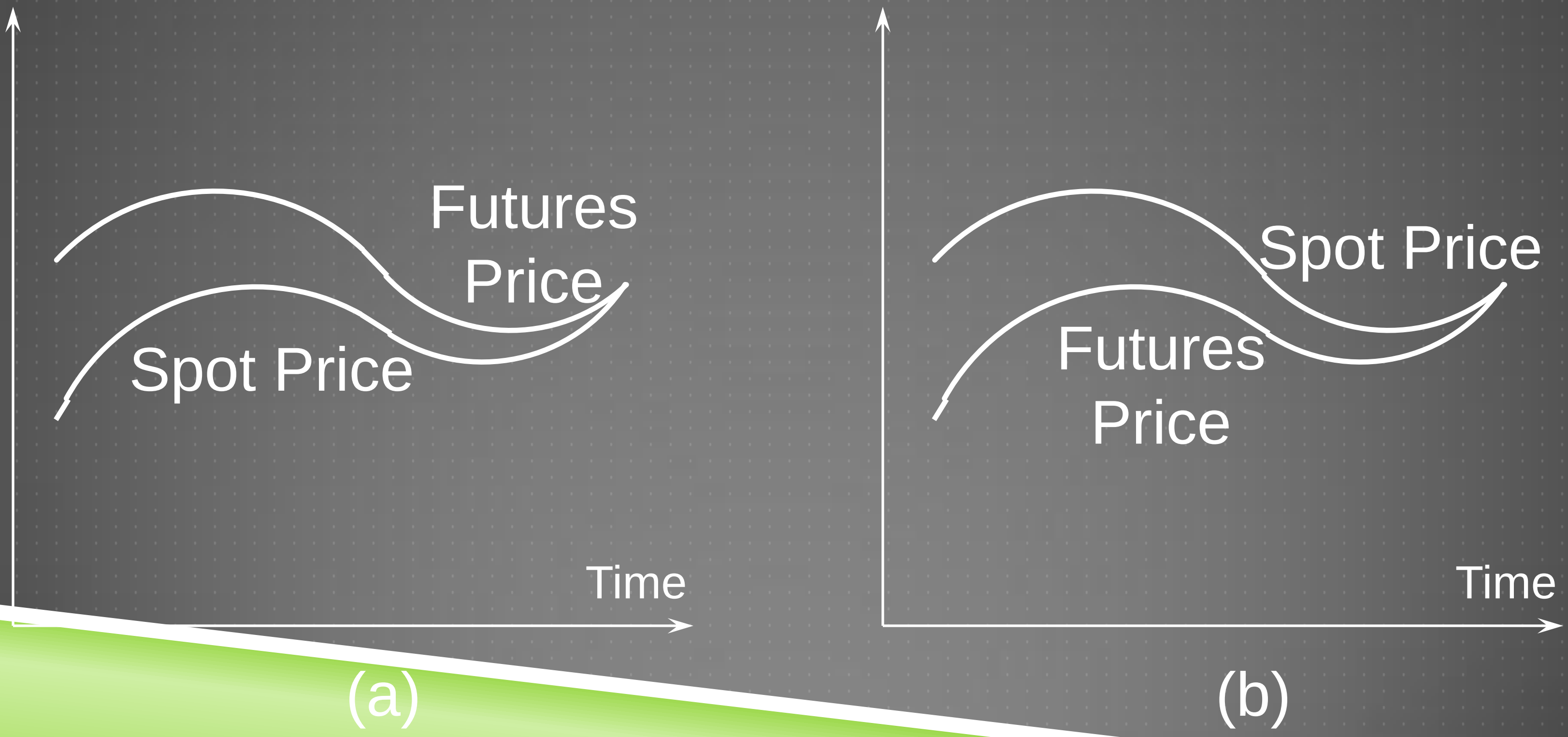
FUTURES ON WHAT?

- Just about anything. "If you can say it in polite company, there is probably a market for it," advertises the CME.
- For example, the CME trades futures on agricultural commodities, foreign currencies, interest rates, and stock market indices, including
  - Agricultural commodities: Live Cattle, Feeder Cattle, Live Hogs, Pork Bellies, Broiler Chickens, Random-Length Lumber.
  - Foreign currencies: Euro, British pound, Canadian dollar, Japanese yen, Swiss franc, Australian dollar, ...
  - Interest rates: Eurodollar, Euromark, 90-Day Treasury bill, One-Year Treasury bill, One-Month LIBOR
  - Stock indices: S&P 500 Index, S&P MidCap 400 Index, Nikkei 225 Index, Major Market Index, FT-SE 100 Share Index, Russell 2000 Index
- Major growth since early '80s has been in financials - now the dominant sector.

DEFINITIONS

- Open Interest: This is the total number of contracts outstanding. It is the sum of all the long positions (or equivalently it is the sum of all the short positions).
- Settlement Price: Usually, this is the average of the prices at which the contract traded immediately before the end of trading for the day.
- High: Highest price in day *t*.
- Low: Lowest price in day *t*.

CONVERGENCE OF FUTURES TO SPOT



HOW DO WE DETERMINE FORWARD/FUTURES PRICES?

- Is there an arbitrage opportunity?
  - The spot price of gold is $300.
  - The 1-year forward price of gold is $340.
  - The 1-year USD interest rate is 5% per annum, continuously compounding.
- Apply the principle of arbitrage:
- The key idea underlying a forward contract is to lock in a price for a security.
- Another way to lock in a price is to buy now and carry the security to the future.
- Since the two strategies have the same effect, they should generate the same P&L. Otherwise, short the expensive strategy and long the cheap strategy.
- The expensive/cheap concept is relative to the two contracts only. Maybe both prices are too high or too low, compared to the fundamental value ...

TWO INTERESTING QUESTIONS

- What is the relationship of the futures price F_{01} to the current spot price S_0?
  - Backwardation/contango.
- What is the relationship of the futures price F_{01} to the future spot price S_1?
  - Risk premium/bias.

FUTURES PRICES & EXPECTED FUTURE SPOT PRICES

- Suppose k is the expected return required by investors on an asset
- We can invest F_0e^{-rT} now to get S_T back at maturity of the futures contract
- This shows that $F_0 = E(S_T)e^{(r-k)T}$
- If the *asset* has
  - no systematic risk, then $k = r$ and F_0 is an unbiased estimate of S_T
  - positive systematic risk, then

    $k > r$ and $F_0 < E(S_T)$
  - negative systematic risk, then

    $k < r$ and $F_0 > E(S_T)$

TWO INTERESTING QUESTIONS (2) (COMMODITY)

- The Expectation Hypothesis of futures prices states that:
  - $F_{0T} = E_0(S_T)$ (risk neutrality assumption).

- Keynes (1930) first proposed that futures prices contain a risk premium (the Expectation Hypothesis does not work)
  - Hedgers are (usually) short futures ==> speculators are long: The only way speculators are willing to be long is if they expect to earn higher returns ==> $F_{0T} < E_0(S_T)$ ==> "normal Backwardation"
  - Normal Contango: $F_{0T} > E_0(S_T)$.

NORMAL BACKWARDATION

- Theory of Normal Backwardation

$$F_0 = E(S_T)e^{(r-k)T}$$

Spot Price	$70.00
Riskless Rate	4%
Time	1
Equity Risk Premium (ERP)	6%
Commodity Beta	0.5
Commodity discount rate	7.00%
Expected Future Spot Price	$ 75.08
Implied Forward Price	$ 72.86
Risk Premium	3.00%
Spot as a function of Premium	$ 75.08

HEDGERS
SHORT FORWARD

SPECULATORS
LONG FORWARD

$E(S_T)$	$ 75.08	
F_0	$ 72.86	
S0	$ 70.00	
Long Forward (Speculators)	$ 2.22	3.0%
Short Forward (Hedgers)	$ (2.22)	

NORMAL BACKWARDATION

- Theory of Normal Backwardation

$$F_0 = E(S_T)e^{(r-k)T}$$

Spot Price	$70.00
Riskless Rate	4%
Time	1
Equity Risk Premium (ERP)	6%
Commodity Beta	0
Commodity discount rate	4.00%
Expected Future Spot Price	$ 72.86
Implied Forward Price	$ 72.86
Risk Premium	0.00%
Spot as a function of Premium	$ 72.86

HEDGERS
SHORT FORWARD

SPECULATORS
LONG FORWARD

$E(S_T)$	$ 72.86	
F_0	$ 72.86	
S0	$ 70.00	
Long Forward (Speculators)	$ -	0.0%
Short Forward (Hedgers)	$ -	

PRICING FORWARD CONTRACTS VIA REPLICATION

- Since signing a forward contract is equivalent (in eect) to buying the security and carry it to maturity.
- The forward price should equal to the cost of buying the security and carrying it over to maturity:

  F(t,T) = S(t) + cost of carry - benefits of carry:

- Apply the principle of arbitrage: Buy low, sell high.
  - The 1-year later (at expiry) cost of signing the forward contract now for gold is $340.
  - The cost of buying the gold now at the spot ($300) and carrying it over to maturity (interest rate cost because we spend the money now instead of one year later) is:

$$S_t\ e^{r(T-t)} = 300\ e^{(0.05*1)} = 315.38$$

- (The future value of the money spent today)
  - Arbitrage: Buy gold is cheaper than signing the contract, so buy gold today and short the forward contract.

CARRYING COSTS

- Interest rate cost: If we buy today instead of at expiry, we endure interest rate cost - In principle, we can save the money in the bank today and earn interests if we can buy it later.
  - This amounts to calculating the future value of today's cash at the current interest rate level.
  - If 5% is the annual compounding rate, the future value of the money spent today becomes, $S_t(1 + r)*1 = 300\ (1 + .05) = 315$
- Storage cost: We assume zero storage cost for gold, but it could be positive...
  - Think of the forward price of live hogs, chicken, ...
  - Think of the forward price of electricity, or weather ...

CARRYING BENEFITS

- Interest rate benefit: If you buy pound (GBP) using dollar today instead of later, it costs you interest on dollar, but you can save the pound in the bank and make interest on pound. In this case, what matters is the interest rate difference:

$$F(t,T)[GBPUSD] = S_t e^{(r_{USD} - r_{GBP})(T-t)}$$

  - In discrete (say annual) compounding, you have something like:

$$F(t;T)[GBPUSD] = S_t(1 + r_{USD})^{(T-t)} =(1 + r_{GBP})^{(T-t).}$$

- Dividend benefiot: similar to interests on pound
  - Let q be the continuously compounded dividend yield on a stock, its forward price becomes, $F(t,T) = S_t e^{(r-q)(T-t).}$
  - The effect of discrete dividends: $F(t,T) = S_t e^{r(T-t)}$- Time-T Value of all dividends received between time t and T
- Also think of piglets, eggs, ...

2. GOLD: ANOTHER ARBITRAGE OPPORTUNITY?

- Suppose that:
  - The spot price of gold is US$300
  - The 1-year forward price of gold is US$300
  - The 1-year US$ interest rate is 5% per annum
- Is there an arbitrage opportunity?

ANOTHER EXAMPLE OF ARBITRAGE

Is there an arbitrage opportunity?

- The spot price of oil is $19
- The quoted 1-year futures price of oil is $25
- The 1-year USD interest rate is 5%, continuously compounding.
- The annualized storage cost of oil is 2%, continuously compounding.

ANOTHER EXAMPLE OF ARBITRAGE

Is there an arbitrage opportunity?

- The spot price of oil is $19
- The quoted 1-year futures price of oil is $25
- The 1-year USD interest rate is 5%, continuously compounding.
- The annualized storage cost of oil is 2%, continuously compounding.

Think of an investor who has oil at storage to begin with.

ANOTHER EXAMPLE OF ARBITRAGE?

Is there an arbitrage opportunity?

- The spot price of electricity is $100 (per some unit...)
- The quoted 3-month futures price on electricity is $110
- The 1-year USD interest rate is 5%, continuously compounding.
- Electricity cannot be effectively stored

How about the case where the storage cost is enormously high?

HEDGING USING FUTURES

- A long futures hedge is appropriate when you know you will purchase an asset in the future and want to lock in the price.
- A short futures hedge is appropriate when you know you will sell an asset in the future and want to lock in the price.
- By hedging away risks that you do not want to take, you can take on more risks that you want to take while maintaining the aggregate risk levels.
  - Companies can focus on the main business they are in by hedging away risks arising from interest rates, exchange rates, and other market variables.
  - Insurance companies can afford to sell more insurance policies by buying re-insurance themselves.
  - Mortgage companies can sell more mortgages by packaging and selling some of the mortgages to the market.

BASIS RISK

- Basis is the difference between spot and futures (S - F).
- Basis risk arises because of the uncertainty about the basis when the hedge is closed out.
- Let (S1; S2; F1; F2) denote the spot and futures price of a security at time 1 and 2.
  - Long hedge: Entering a long futures contract to hedge future purchase:

$$\text{Future Cost} = S2 - (F2 - F1) = F1 + \text{Basis}:$$

  - Short hedge: Entering a short futures contract to hedge future sell:

$$\text{Future Prot} = S2 - (F2 - F1) = F1 + \text{Basis}:$$

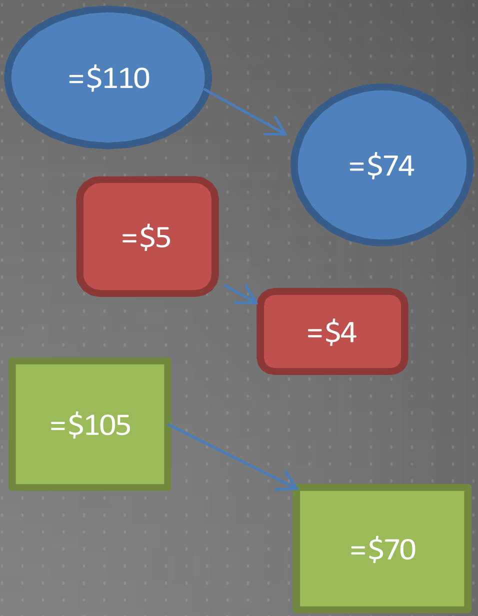
BASIS RISK

- BASIS RISK

Basis Risk
Crude Oil

	Sep-08	Sep-09
Spot	110	74
Futures	105	70
Basis	5	4

Spot	$ (74.00)
Futures (gain/loss)	$ (35.00)
Total Cost	$ (109.00)



Long Hedge

Unexpected basis strengthening: LOSS
Unexpected basis weaking: PROFIT

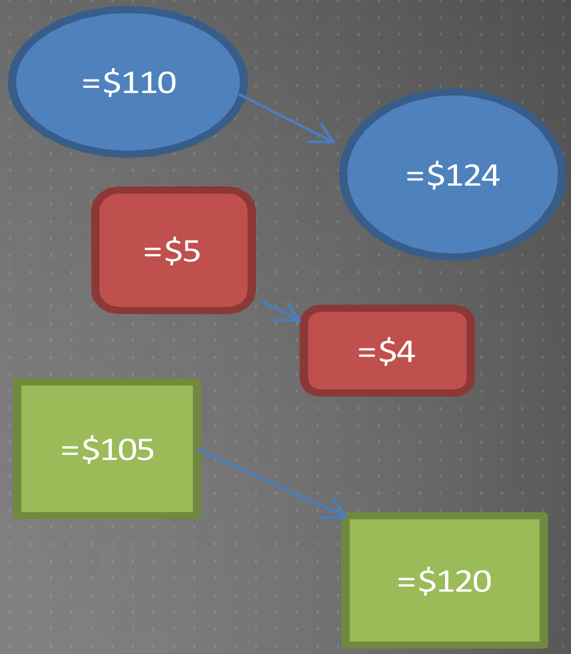
BASIS RISK

- BASIS RISK

Basis Risk
Crude Oil

	Sep-08	Sep-09
Spot	110	124
Futures	105	120
Basis	5	4

Spot	$ (124.00)
Futures (gain/loss)	$ 15.00
Total Cost	$ (109.00)



Long Hedge

Unexpected basis strengthening: LOSS
Unexpected basis weaking: PROFIT

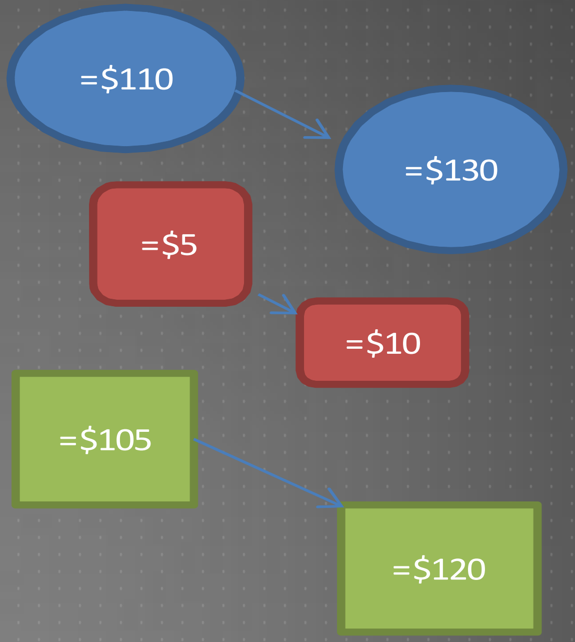
BASIS RISK

- BASIS RISK: Basis Strengthening

Basis Risk
Crude Oil

	Sep-08	Sep-09
Spot	110	130
Futures	105	120
Basis	5	10

Spot	$ (130.00)
Futures (gain/loss)	$ 15.00
Total Cost	$ (115.00)



Long Hedge

Unexpected basis strengthening: LOSS
Unexpected basis weaking: PROFIT

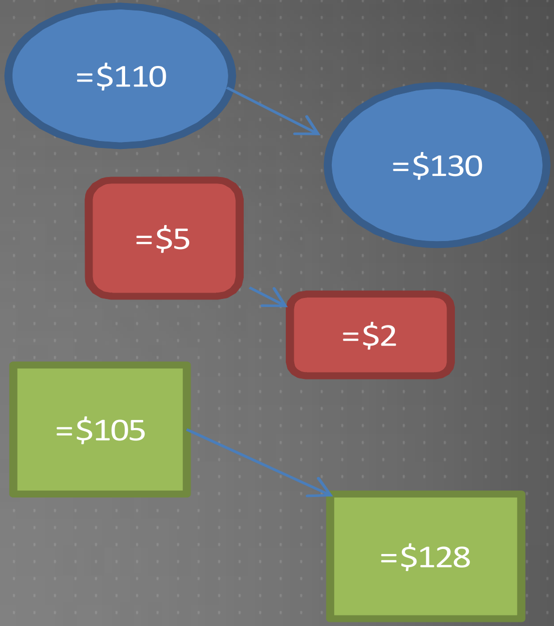
BASIS RISK

- Basis Risk: Basis Weakening

Basis Risk
Crude Oil

	Sep-08	Sep-09
Spot	110	130
Futures	105	128
Basis	5	2

Spot	$ (130.00)
Futures (gain/loss)	$ 23.00
Total Cost	$ (107.00)



Long Hedge

Unexpected basis strengthening: LOSS
Unexpected basis weaking: PROFIT

CHOICE OF CONTRACT

- Choose a delivery month that is as close as possible to, but later than, the end of the life of the hedge
- When there is no futures contract on the asset being hedged, choose the contract whose futures price is most highly correlated with the asset price. This is known as cross hedging.

OPTIMAL HEDGE RATIO

- For each share of the spot security, the optimal share on the futures (that minimizes future risk) is:

Proportion of the exposure that should optimally be hedged is

$$h^* = \rho \frac{\sigma_S}{\sigma_F}$$

where

σ_S is the standard deviation of ΔS, the change in the spot price during the hedging period,

σ_F is the standard deviation of ΔF, the change in the futures price during the hedging period

ρ is the coefficient of correlation between ΔS and ΔF.

OPTIMAL NUMBER OF CONTRACTS

$$N^* = \frac{h^* Q_A}{Q_F}$$

Where

N^* : optimal number of futures contracts for hedging

h^* : hedge ratio that minimizes the variance of the hedger’s position

Q_A : size of position being hedged (units)

Q_F : size of one futures contract (units)

EXAMPLE

- An airline expects to purchase 2 million gallons of jet fuel in 1 month and decides to use heating oil for hedging. Assume that volatility of jet fuel in the last 15 months is 2.63% and heating oil is 3.13%. Correlation between heating oil and jet fuel price change is 0.928. Each heating oil contract traded on NYMEX is on 42000 gallons of heating oil. Calculate minimum hedge ratio and optimal number of contracts for hedging.

$$h^* = \rho \frac{\sigma_S}{\sigma_F} = 0.928 \times \frac{0.0263}{0.0313} = 0.78$$

$$N^* = \frac{h^* Q_A}{Q_F} = \frac{0.78 \times 2,000,000}{42,000} = 37.14$$

If airline buys 37 NYMEX heating oil contracts, airline will be hedged optimally.

TAILING THE HEDGE

- Two way of determining the number of contracts to use for hedging are
  - Compare the exposure to be hedged with the value of the assets underlying one futures contract
  - Compare the exposure to be hedged with the value of one futures contract (=futures price time size of futures contract
- The second approach incorporates an adjustment for the daily settlement of futures. In practice this means that

$$N^* = \frac{h^* V_A}{V_B}$$

where V_A dollar value of the position being hedged and V_B dollar value of one futures contract. If we assume spot price of jet fuel is 1.94 and futures pric of heating oil is 1.99, then optimal number of contract is about 36 contracts.

REGRESSIONS ON RETURNS

- A simple way to obtain the optimal hedge ratio is to run the following least
- square regression:
- $\Delta S = a + b\Delta F + e$
  - b is the optimal hedge ratio estimate for each share of the spot.
  - The variance of the regression residual (e) captures the remaining risk of the hedged position ($\Delta S - a - b \Delta F$).
- Many times, we estimate the correlation or we run the regressions on returns instead of on price changes for stability:

$$\frac{\Delta S}{S} = \alpha + \beta \frac{\Delta F}{F} + \varepsilon$$

  - Comparing β from the return regression with the optimal hedging ratio in the price change regression, we need to adjust for the value (scale) difference to obtain the hedging ratio in shares: $b = \beta S/F$.
  - Example: Hedge equity portfolios using index futures based on CAPM

HEDGING USING INDEX FUTURES

- To hedge the risk in a portfolio the number of contracts that should be shorted is

$$\beta P/F$$

where P is the value of the portfolio, β is its beta, and F is the value of one futures contract

EXAMPLE

S&P 500 futures price is 1,000

Value of Portfolio is $5 million

Beta of portfolio is 1.5

What position in futures contracts on the S&P 500 is necessary to hedge the portfolio?

CHANGING BETA

- What position is necessary to reduce the beta of the portfolio to 0.75?
- What position is necessary to increase the beta of the portfolio to 2.0?

HEDGING PRICE OF AN INDIVIDUAL STOCK

- Similar to hedging a portfolio
- Does not work as well because only the systematic risk is hedged
- The unsystematic risk that is unique to the stock is not hedged

WHY HEDGE EQUITY RETURNS

- May want to be out of the market for a while. Hedging avoids the costs of selling and repurchasing the portfolio
- Suppose stocks in your portfolio have an average beta of 1.0, but you feel they have been chosen well and will outperform the market in both good and bad times. Hedging ensures that the return you earn is the risk-free return plus the excess return of your portfolio over the market.

ROLLING THE HEDGE FORWARD

- We can use a series of futures contracts to increase the life of a hedge
- Each time we switch from one futures contract to another we incur a type of basis risk

HEDGE FUNDS

- Hedge funds are not subject to the same rules as mutual funds and cannot offer their securities publicly.
- Mutual funds must
  - disclose investment policies,
  - makes shares redeemable at any time,
  - limit use of leverage
  - take no short positions.
- Hedge funds are not subject to these constraints.
- Hedge funds use complex trading strategies are big users of derivatives for hedging, speculation and arbitrage