INTEREST RATES

INTEREST RATE

- An interest rate is the amount of money a borrower promises to pay the lender. Depends on
  - Inflation
  - Risk premium (credit default risk)

TYPES OF RATES

- Treasury rates (the rates an investor earns on Treasury bills or bonds)
- LIBOR (London Interbank Offered Rate) rates: rate of interest at which the bank or other financial institutions is prepared to make a large wholesale deposits with other banks.
  - LIBID (London Interbank Bid Rate) the rate at which the bank will accept deposits from other banks.
- Repo (Repurchasing Agreement) rates: The price at which securities are sold and the price at which they are repurchased is referred to as repo rate.

MEASURING INTEREST RATES

- The compounding frequency used for an interest rate is the unit of measurement
- The difference between quarterly and annual compounding is analogous to the difference between miles and kilometers

$$FV_t = A(1+R)^n \quad Compounded\ once\ per\ annum$$

$$FV_t = A(1+\frac{R}{m})^{mn} \quad Compounded\ m\ times\ per\ annum$$

CONTINUOUS COMPOUNDING (PAGE 77)

- In the limit as we compound more and more frequently we obtain continuously compounded interest rates $V_t = Ae^{Rn}$
- \$100 grows to \$$100e^{RT}$ when invested at a continuously compounded rate R for time T
- \$100 received at time T discounts to \$$100e^{-RT}$ at time zero when the continuously compounded discount rate is R

MEASURING INTEREST RATE

- Effect of the compounding frequency on the value of $1000 at the end of 10 year when the interest rate is 5% per year

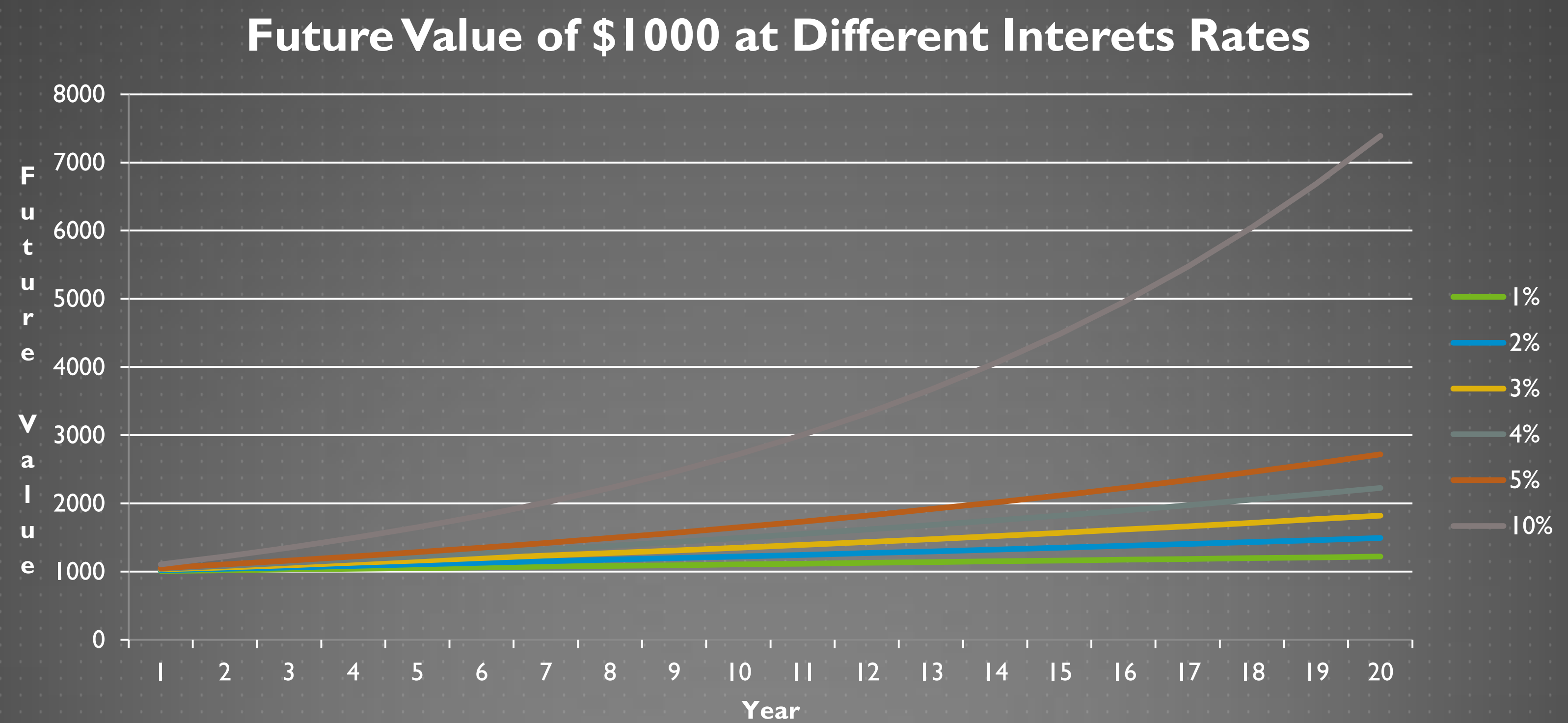
Compounding frequency	Value of $1000 at the end of 10 year
Annually (m=1)	1628.895
Semi-annual (m=2)	1643.616
Quarterly (m=4)	1643.619
Monthly (m=12)	1647.009
Weekly (m=52)	1648.325
Daily (m=365)	1648.665
Continuous	1648.721

EFFECT OF COMPOUNDING FREQUENCY

- Effect of compounding frequency: How much you should invest in order to get $1000 at the end of 10 year when the interest rate is 5% per year

Annually (m=1)	613.9132535
Semi-annual (m=2)	610.2709429
Quarterly (m=4)	608.4133355
Monthly (m=12)	607.1610403
Weekly (m=52)	606.6763845
Daily (m=365)	606.5514298
Continuous	606.5306597

FUTURE VALUE OF MONEY



FUTURE VALUE AND INTEREST EARNED

- Future Value and Interest Earned

Year	Beginning Amount	Interest Earned	Ending Amount
1	$1,000.00	$105.17	$1,105.17
2	$1,105.17	$116.23	$1,221.40
3	$1,221.40	$128.46	$1,349.86
4	$1,349.86	$141.97	$1,491.82
5	$1,491.82	$156.90	$1,648.72
6	$1,648.72	$173.40	$1,822.12
7	$1,822.12	$191.63	$2,013.75
8	$2,013.75	$211.79	$2,225.54
9	$2,225.54	$234.06	$2,459.60
10	$2,459.60	$258.68	$2,718.28
11	$2,718.28	$285.88	$3,004.17
12	$3,004.17	$315.95	$3,320.12
13	$3,320.12	$349.18	$3,669.30
14	$3,669.30	$385.90	$4,055.20
15	$4,055.20	$426.49	$4,481.69
16	$4,481.69	$471.34	$4,953.03
17	$4,953.03	$520.91	$5,473.95
18	$5,473.95	$575.70	$6,049.65
19	$6,049.65	$636.25	$6,685.89
20	$6,685.89	$703.16	$7,389.06
Total Interest Earned		$6,389.06	

FREQUENCY OF COMPOUNDING

- Interest rates are usually stated in the form of an *annual percentage rate* with a certain frequency of compounding. Since the frequency of compounding can differ, it is important to have a way of making interest rates comparable. This is done by computing *effective annual rate* (EFF), defined as the equivalent interest rate, if compounding were only once per year.

$$EFF = (1 + \frac{R}{m})^m - 1$$

CONVERSION FORMULAS (PAGE 77)

- What if we want to find the equivalent interest rate, if compounding is done continuously?

Define

R_c : continuously compounded rate

R_m: equivalent rate with compounding *m* times per year

$$R_c = m \ln\left(1 + \frac{R_m}{m}\right)$$

$$R_m = m\left(e^{R_c/m} - 1\right)$$

PURE DISCOUNT BONDS (ZERO-COUPON BONDS)

A zero rate (or spot rate), for maturity *T* is the rate of interest earned on an investment that provides a payoff only at time *T*

- Discount bonds, also called zero-coupon bonds, are securities which "*make a single payment at a date in the future known as maturity date. The size of this payment is the face value of the bond. The length of time to the maturity date is the maturity of the bond*" (Campbell, Lo, MacKinley (1996)).

PURE DISCOUNT BOND

- The promised cash payment on a pure discount bond is called its *face value* or *par value*. Yield (interest rate) on a pure discount bond is the annualized rate of return to investors who buy it and hold it until it matures.

$$\text{Yield on 1 - year pure discount bond} = \frac{FV - Price}{Price}$$

EXAMPLE

Maturity (years)	Zero Rate (% cont comp)
0.5	5.0
1.0	5.8
1.5	6.4
2.0	6.8

BOND PRICING

- To calculate the cash price of a bond we discount each cash flow at the appropriate zero rate

- The theoretical price of a two-year bond providing a 6% coupon semiannually is

$$3e^{-0.05\times 0.5} + 3e^{-0.058\times 1.0} + 3e^{-0.064\times 1.5}$$
$$+ 103e^{-0.068\times 2.0} = 98.39$$

BOND YIELD

- The bond yield is the discount rate that makes the present value of the cash flows on the bond equal to the market price of the bond
- Suppose that the market price of the bond in our example equals its theoretical price of 98.39
- The bond yield is given by solving

$$3e^{-y\times 0.5}+3e^{-y\times 1.0}+3e^{-y\times 1.5}+103e^{-y\times 2.0}=98.39$$

to get y = 0.0676 or 6.76%.

PAR YIELD

- The par yield for a certain maturity is the coupon rate that causes the bond price to equal its face value.
- In our example we solve

$$\frac{c}{2}e^{-0.05\times0.5}+\frac{c}{2}e^{-0.058\times1.0}+\frac{c}{2}e^{-0.064\times1.5}$$

$$+\left(100+\frac{c}{2}\right)e^{-0.068\times2.0}=100$$

to get c=6.87 (with s.a. compounding)

PAR YIELD (CONTINUED)

In general if *m* is the number of coupon payments per year, *d* is the present value of $1 received at maturity and *A* is the present value of an annuity of $1 on each coupon date

$$c = \frac{(100 - 100d)m}{A}$$

BOOTSTRAP METHOD TO CALCULATE DISCOUNT FACTOR

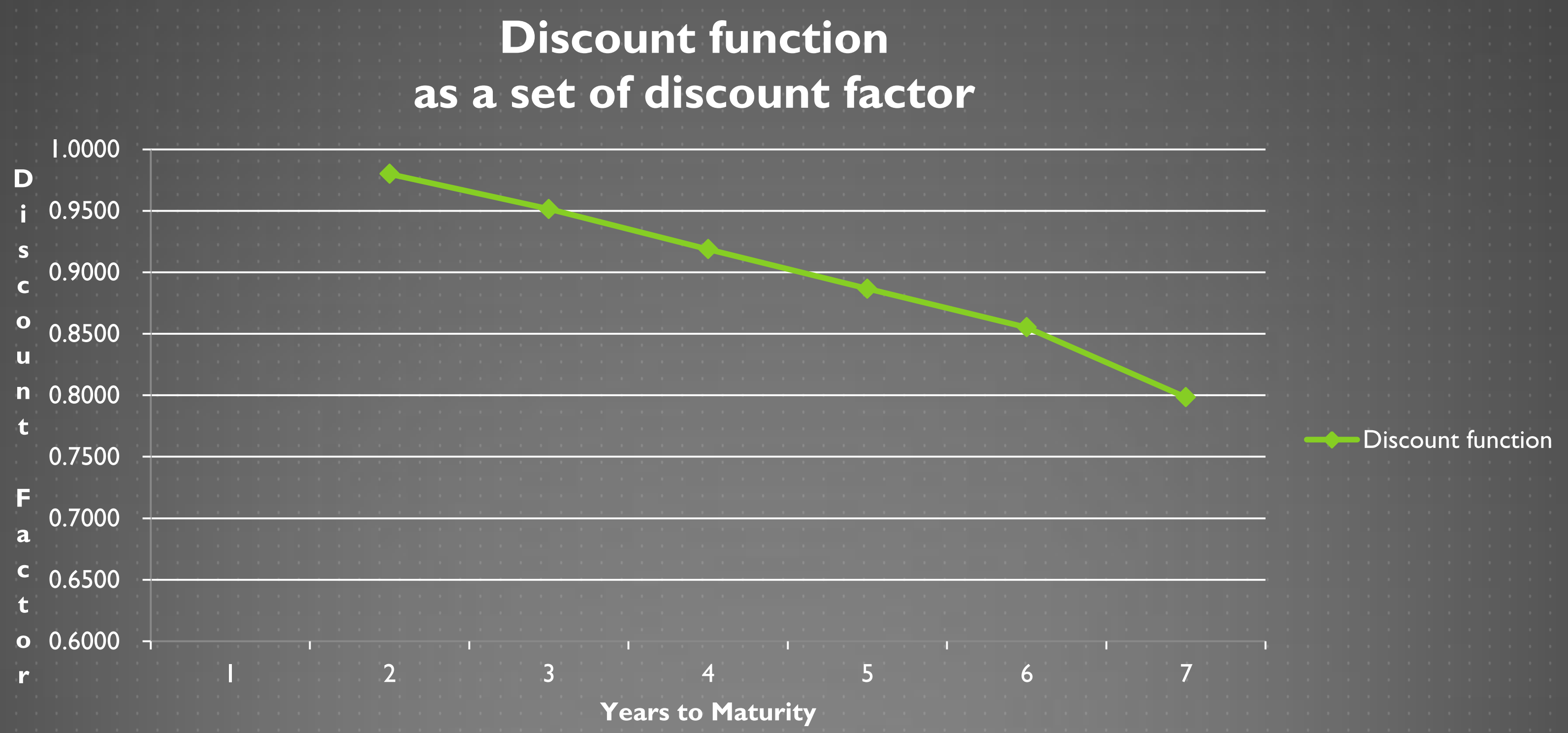
- A discount function is a set of discount factors, where each discount factor is just a present value multiplier. For example, d(1.0) is the present value of $1 dollar received in one year. The key idea is that each d(x) can be solved as one variable under one equation because we already solved for shorter-term discount factors.
- The most popular approach is to use *bootstrap method*

BOOTSTRAP : EXAMPLE

d(.5) is present value of $1 in six month
d(1) is present value of $1 in one year
$980=d(.5)*($1000 in six month)

Face (par) value		$1,000					
Years to Maturity		0.5	1	1.5	2	2.5	3
Bond Price		$980	$990	$990	$980	$970	$960
Coupon Rate		0.00%	4.00%	5.00%	5.00%	5.00%	6.00%
Discount function		0.9800	0.9514	0.9187	0.8866	0.8552	0.7983
FV Cash Flows							
Years	0.5	$1,000	$20	$25	$25	$25	$30
	DF	0.9800	0.9800	0.9800	0.9800	0.9800	0.9800
	1		$1,020	$25	$25	$25	$30
	DF		0.9514	0.9514	0.9514	0.9514	0.9514
	1.5			$1,025	$25	$25	$30
	DF			0.9187	0.9187	0.9187	0.9187
	2				$1,025	$25	$30
	DF				0.8866	0.8866	0.8866
	2.5					$1,025	$30
	DF					0.8552	0.8552
	3						$1,030
							0.7983

DISCOUNT FACTOR



DETERMINING TREASURY ZERO RATES

Face (par) value		$1,000				
Years to Maturity		0.25	0.5	1	1.5	2
Bond Price		$975	$949	$900	$960	$1,016
Coupon Rate		0.00%	0.00%	0.00%	8.00%	12.00%
Discount function		0.9750	0.9490	0.9000	0.8520	0.8056
	Zero Rate:	10.127%	10.469%	10.536%	10.681%	10.808%
FV Cash Flows						
Years	0.25	$1,000	$0	$0	$0	$0
	DF	0.9750	0.9800	0.9800	0.9800	0.9800
	0.5		$1,000	$0	$40	$60
	DF		0.9490	0.9490	0.9490	0.9490
	1			$1,000	$40	$60
	DF			0.9000	0.9000	0.9000
	1.5				$1,040	$60
	DF				0.8520	0.8520
	2					$1,060
	DF					0.8056

TREASURY ZERO RATE CURVE