DETERMINATION OF FORWARD AND FUTURES PRICES

CONSUMPTION VS INVESTMENT ASSETS

- Investment assets are assets held by significant numbers of people purely for investment purposes (Examples: gold, silver)
- Consumption assets are assets held primarily for consumption (Examples: copper, oil)

SHORT SELLING

- Short selling involves selling securities you do not own
- Your broker borrows the securities from another client and sells them in the market in the usual way
- At some stage you must buy the securities back so they can be replaced in the account of the client
- You must pay dividends and other benefits the owner of the securities receives

DETERMINATION OF FUTURES PRICES

- Futures prices are determined competitively by equality of supply and demand.
- However, we can often use arbitrage arguments to obtain simple expressions.
- These arguments are simplified by ignoring differences between forward and futures prices - the two are the same if the rate of interest is constant:
  - Cox, Ingersoll and Ross (1981), "The Relation between Forward Prices and Futures Prices", *Journal of Financial Economics*, 9, 321 - 346.
  - Jarrow and Oldfield (1981), "Forward Contracts and Futures Contracts", *Journal of Financial Economics*, 9, 373 - 382.

THE ARBITRAGE PRINCIPLE

- If two strategies, *A* and *B*, have identical returns in all states of the world, they must cost the same.
- Suppose they had different costs, $C_A > C_B$: one may short *A* and long *B* to make an initial profit of $C_A - C_B$ instantly and risklessly.
- The arbitrage principle requires:
  - no transaction costs
  - no restrictions on short selling
  - the market participants can borrow money at the same risk-free rate of interest as they can lend money.

ALTERNATIVE WAYS TO BUY STOCKS

- Outright Purchase: Simultaneously pay the stock price and receive ownership of the stock
- Fully Leveraged Purchase: A purchase in which you borrow the entire purchase price of the security.
- Prepaid forward Contract: An arrangement in which you pay for the stock today and receive the stock at an agreed-upon future date.
- Forward Contract: An arrangement in which you both pay for the stock and receive it at an agreed-upon date and price.

ALTERNATIVE WAYS TO BUY STOCK

- Example: Stock price at time 0 is S_0

Description	Pay at time	Receive Security at time	Payment
Outright	0	0	S_0 at time 0
Fully Leveraged	T	0	S_0e^{rT}
Prepaid Contract	0	T	?
Forward Contract	T	T	?*e^{rT}

- Difference is the time to take the ownership. If you defer payment, you should pay interest.

PREPAID FORWARD CONTRACTS ON STOCK

- A prepaid forward contract entails paying today to receive ownership at a future date. The sale of a prepaid forward contracts permits the owner to sell an asset while retaining the ownership of the asset for a period of time. Buyer of the contract will get the ownership in the future. If there is no dividend payments, then price of the prepaid contract is the stock price today.

$$F_{0,T}^{P} = S_0$$

PREPAID FORWARD CONTRACT

- Or alternatively we can derive the prepaid forward contract price using present value.

$$F^P_{0,T} = E_0(S_T)e^{-rT}$$

$$E_0(S_T) = S_0e^{rT}, F^P_{0,T} = S_0e^{rT}e^{-rT}, F_{0,T} = S_0$$

- For a non-dividend paying stock, the prepaid forward price is the stock price.

PRICING PREPAID FORWARD CONTRACT BY ARBITRAGE

- Arbitrage is defined as profit by simultaneously buying and selling related asset.
- Since there are many market participants, we should not see any arbitrage opportunities (risk-free profit).
  - Assume that price of stock in future will be S_T
  - Assume that prepaid forward contract price is higher than spot price $F_{0,T} > S_0$. Arbitrageur will buy low and sell high by buying the stock for S_0 and selling the prepaid contract for $F_{0,T}$
  - Assume that prepaid forward contract price is lower than spot price $F_{0,T} < S_0$. Arbitrageur will buy low and sell high by shorting the stock for S_0 and buying the prepaid contract for $F_{0,T}$

PRICING PREPAID FORWARD WITH DIVIDEND

- In the case of discrete dividends

$$F^P_{0,T} = S_0 - \sum_{i=1}^{n} PV_{0,t_i}(D_{t_i})$$

- In the case of continuous dividends

where q is the dividend yield $F^P_{0,T} = S_0 e^{-qT}$

FORWARDS CONTRACTS ON STOCK

- The only difference between prepaid forward contract and forward contract is the timing of the payment for the stock. Thus, the forward price is just the future value of the prepaid forward.

$$F_{0,T} = S_0 e^{rT}$$

  - If $F_0 > S_0 e^{rT}$, then arbitrageurs can buy the asset and short forward contract on the asset.
  - If $F_0 < S_0 e^{rT}$, then arbitrageurs can short the asset and buy forward contract on the asset.
- If the stock pays discrete dividends, then the forward price will be

$$F_{0,T} = S_0 - \sum_{i=1}^{n} e^{r(T-t_i)} D_{t_i}$$

FORWARDS CONTRACTS ON STOCK

- If the stock pays continuous dividends, then the forward price will be

Where q is the dividend $F_{0,T} = S_0 e^{(r-q)T}$ ee rate.

  - The forward price is higher than spot price because of the cost of financing the spot purchase of the asset during the life of the forward contract.
- When an investment asset will provide income with a present value of I during the life of a forward contract, we have

$$F_{0,T} = (S_0 - I)e^{rT}$$

FORWARD PRICE AND FORWARD PREMIUM

- Since the has deferred payment, its initial premium is zero, it is initially costless.
- The forward price, however, is the future value of the prepaid forward price.
- Forward premium can be defined as the ratio of forward price to the spot price

$$Forward \;\; premium = \frac{F_0, T}{S_0}$$

VALUING FORWARD CONTRACTS

- However, it is important to value the contract each day. Assume that K is the delivery price that was negotiated some time ago, $F_{0,T}$ is the forward price that would be applicable if we negotiated today, and f is the value of the contract today.
- Remember, at the beginning of the life of the contract K= $F_{0,T}$ and f=0.

FORWARD PRICE AND FORWARD PREMIUM

- K always stays same since that is the delivery price, but forward price changes. In general, for long forward contract

$$f = (F_0 - K)e^{-rT}$$

- And for all short forward contract

$$f = (K - F_0)e^{-rT}$$

DOES FORWARD PRICE PREDICT THE FUTURE PRICE?

- It is common to think that the forward price reflects an expectation of the asset's future price.
- However forward price conveys no additional information about the expected future spot price
- Moreover, the forward price systematically errs in predicting the future spot price.
- When you buy stock
  - You invest money that has a opportunity cost (r)
  - You are acquiring the risk of the stock (α-r), where α is expected rate of return.

DOES FORWARD PRICE PREDICT THE FUTURE PRICE?

- When you enter into a forward contract, there is no investment, so there is no opportunity cost of money invested
- However, the forward contract retains the risk of the contract, you must be compensated for risk.
- This means that the forward contract must earn risk premium. If the risk premium is positive, you expect a positive return from forward contract. The only way this can happen is if the forward price predicts too low a stock price.
- In other words, the forward contract is a biased predictor of the future stock price.

DOES FORWARD PRICE PREDICT THE FUTURE PRICE?

- Let α be the expected return and r be the effective annual interest rate

$$F_0 = S_0(1+r)$$

$$E_0(S_1) = S_0(1+\alpha)$$

$$E_0(S1) - F_0 = S_0(1+\alpha - S_0(1+r) = S_0(\alpha - r)$$

- The expression (α-r) is the risk premium on the asset. As long as risk premium is different from zero, expected future price will be different from future price.

FORWARD PRICING FORMULA

- Forward pricing formula for a stock index

$$F_0 = S_0 e^{(r-q)T}$$

- Depends on (r-q). This difference is called ***cost of carry.***

- Forward Price=Spot Price +Interest to carry asset –asset lease rate

Cost of Carry

FORWARD AND FUTURES CONTRACTS ON CURRENCIES

- Currency futures and forwards contracts are widely used to hedge against changes in exchange rates.
- Many corporations use currency futures and forwards for short-term hedging.

FORWARD EXCHANGE RATE (1)

- Definitions:
  - $ base currency
  - risk-free dollar rate = r
  - risk-free foreign rate = r_f
  - time to maturity = T
  - spot exchange rate = S_0 (Home currency/unit of foreign currency)
  - forward exchange rate = F_{0T}.
- Consider two strategies for owning 1 unit of foreign currency at T:
- Strategy A: Buy 1 unit foreign currency forward at 0 at exchange rate F_{0T}, deposit e^{-rT}/F_{0T} in dollar risk-free asset, use proceeds to pay for currency at T. Cost = e^{-rT}/ F_{0T}.
- Strategy B: Buy $e^{-r_f T}$ units foreign currency at 0 at exchange rate S_0, invest currency in foreign risk-free asset. Cost = $e^{-r_f T}/ S_0$.

FORWARD EXCHANGE RATE (2)

- By Arbitrage Principle, the two strategies must cost the same since they both give the same terminal value S_T. Hence

$$e^{-r_f T}/S_0 = e^{-rT}/F_{0,T}$$

$$F_{0,T} = e^{(r-r_f)T} S_0$$

$$f - s = r - r_f$$

- This is the *Covered Interest Parity* (CIP) condition: if the domestic interest rate exceeds the dollar rate ($r>r_f$), the domestic currency will be seen as depreciating ($F_{0T} > S_0$). The only reason US pays higher interest is the fact that US currency is expected to depreciate. To compensate US $ holder, US pays higher interest rate.

FOREIGN EXCHANGE RATE

- Example: Suppose that euro-denominated interest rate is 3% and dollar-denominated rate is 1%. The current exchange rate is 1.47 dollar per euro. The one-year forward rate

$$1.47e^{0.01-0.03} = 1.44$$

- Thus if $r<r_f$, forward dollar will go to a premium. This is indeed likely to happen as covered arbitrage funds will be flowing through the spot market to the foreign market, where interest rate is higher- in order to buy foreign treasury bills – and this will tend to reduce the price of the dollar in the spot market, S rises, while in the forward market, the buying of forward dollar will strengthen the forward dollar, i.e. F reduce. With S rising while F is falling it is expected that forward premium on the dollar become positive- the dollar is at a premium.

COMMODITY FORWARDS AND FUTURES

- On any given day, for many commodities there are futures contract available that expire in a number of different months.
- The set of prices for different expiration dates for a given commodity is called the forward curve or forward strip.

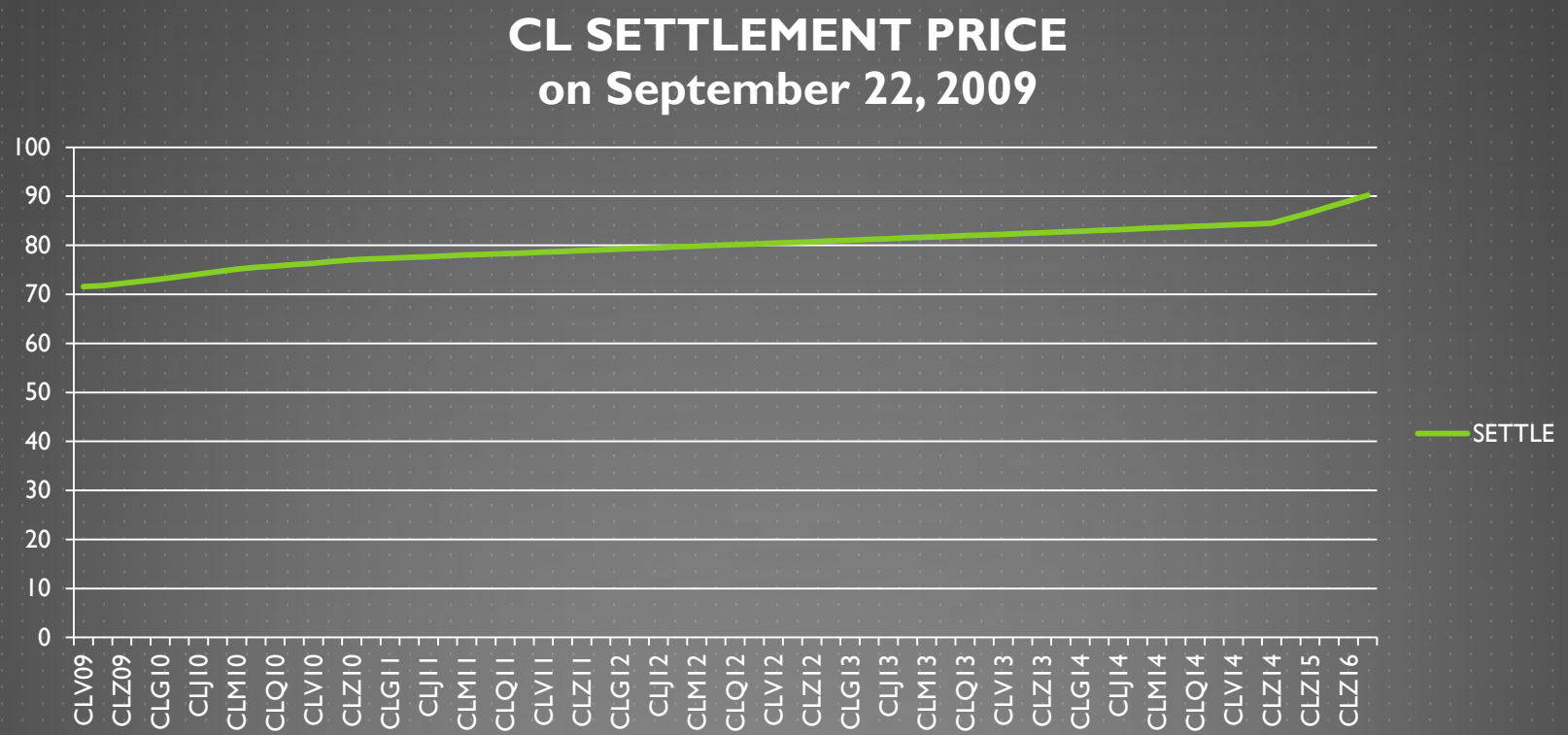
FUTURES MONTHS ABBREVIATION

Symbol	Month
F	January
G	February
H	March
J	April
K	May
M	June
N	July
Q	August
U	September
V	October
X	November
Z	December

CL: LIGHT SWEET OIL

CONTRAC	OPEN	HIGH	LOW	LAST	SETTLE	PRIOR SET	PRIOR INT	TRADEDATE
CLV09	69.67	71.85	69.61	71.5	71.55	69.71	19771	9/22/2009
CLX09	69.77	72.03	69.74	71.62	71.76	69.93	312113	9/22/2009
CLZ09	70.21	72.47	70.21	72.03	72.22	70.41	171554	9/22/2009
CLF10	70.77	72.94	70.77	72.59	72.69	70.94	59495	9/22/2009
CLG10	71.7	73.21	71.7	73.16	73.13	71.43	27762	9/22/2009
CLH10	72.65	73.81	72.65	73.5	73.62	71.94	31626	9/22/2009
CLJ10	72.67	74.16	72.67	74.14	74.11	72.44	15812	9/22/2009
CLK10	74.51	74.61	74.5	74.61	74.61	72.95	15579	9/22/2009
CLM10	73.6	75.29	73.6	75	75.12	73.46	79822	9/22/2009
CLN10	74.79	75.55	74.78	75.55	75.49	73.86	29401	9/22/2009
CLQ10	75.6	75.6	75.4	75.4	75.77	74.17	11067	9/22/2009
CLU10	76.07	76.07	76.07	76.07	76.05	74.5	19380	9/22/2009
CLV10	76.2	76.2	76.2	76.2	76.36	74.84	9553	9/22/2009
CLX10	76.68	76.68	75.97	75.97	76.7	75.21	14252	9/22/2009
CLZ10	75.65	77.26	75.65	77.04	77.04	75.58	102896	9/22/2009

FORWARD CURVE: CL



FORWARD CURVE

- Let us assume 1 year LIBOR rate 1.27% on September 22, 2009. From October 2009 to October 2010, CL price rose from 71.55 to 76.36 or 6.72%, far in excess of 1.27%. In the context of the formula for pricing financial forwards, we would need to have continuous dividend yield, q, of -5.23% in order to explain this rise in the forward price over time. In that case we would have

- How do we interpret negative dividend?

$$F_{September,2011} = 71.55e^{0.0127-(-0.0523)} = 76.36$$

COMMODITIES

- Even more puzzling, sometimes we can see future prices will decline with time to expiration.
- It is possible to explain this behaviour by specific commodity market, e.g. harvesting time in corn or war in Iraq reduce the supply of oil in spot market but expectation for future is more optimistic.
- The behaviour of forward prices can vary over time. When forward curve is upward sloping then we say the market is in contango. If the forward curve is downward sloping then we say market is in backwardation.

FORWARD COMMODITY PRICES (I)

- Definitions: as before plus
  - Storage costs (they can be regarded as negative income) = u (continuos rate)
- Gold and Silver:
  - $F_{0T} = S_0 e^{rT}$ (if u = 0), Gold and Silver can be considered as securities paying no income
  - $F_{0T} = S_0 e^{(r+u)T}$ (if $u \neq 0$).
- Other Commodities:
  - In this case we can only establish an inequality: $F_{0T} \leq S_0 e^{(r+u)T}$.

FORWARD COMMODITY PRICES (2)

- Suppose $F_{0T} > S_0 e^{(r+u)T}$.
  - Sell 1 unit commodity forward at F_{0T}
  - Buy e^{uT} units spot at S_0 and pay for the storage costs
  - Riskless profit = $e^{-rT} F_{0T} - e^{uT} S_0 = e^{-rT}(F_{0T} - e^{(r+u)T} S_0)$.
  - It implies that $F_{0T} \leq S_0 e^{(r+u)T}$.
- Suppose $F_{0T} < S_0 e^{(r+u)T}$.
  - This suggests buying forward and selling spot
  - Industrial users (they are reluctant to sell commodity and buy futures contracts since futures contracts cannot be consumed).
- All we can assert for commodity is: $F_{0T} \leq S_0 e^{(r+u)T}$.

CONVENIENCE YIELD

- Business reasons for holding commodities: nonmonetary return that is sometimes referred to as convenience yield.

$$F_0 = S_0 e^{(r+u-y)T}$$

where y represent the convenience yield.

- The convenience yield reflects the market's expectations concerning the availability of the commodity. The greater the possibility that shortages will occur, the higher the convenience yield.

FORWARD VS FUTURES PRICES

- Forward and futures prices are usually assumed to be the same. When interest rates are uncertain they are, in theory, slightly different

COMMODITY SPREADS

- Some commodities are inputs of other commodities, which gives rise to commodity spreads. Soybeans, for example, can be crushed to produce soybean meal and soybean oil. A trader with a position in soybeans and opposite position in equivalent quantities of soybean oil has a crush spread and is said to be trading the crush.
- Similarly, crude oil is refined to make petroleum products, in particular heating oil and gasoline. The split of oil into these different components can be complemented by a process known as "cracking", hence the difference in price between crude oil and equivalent amounts of heating oil and gasoline is called crack spread.